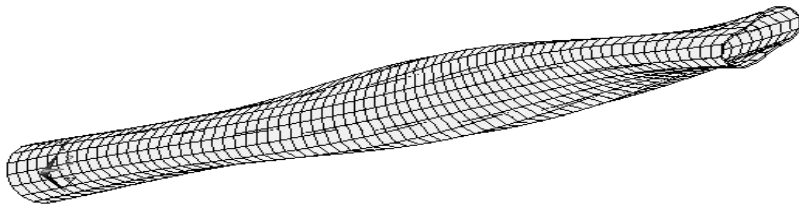




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Optimum Design and Analysis of a Composite Drive Shaft for an Automobile



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Abstract:

Substituting composite structures for conventional metallic structures has many advantages because of higher specific stiffness and strength of composite materials. This work deals with the replacement of conventional two-piece steel drive shafts with a single-piece e-glass/epoxy, high strength carbon/epoxy and high modulus carbon/epoxy composite drive shaft for an automotive application. The design parameters were optimized with the objective of minimizing the weight of composite drive shaft. The design optimization also showed significant potential improvement in the performance of drive shaft.

Keywords:

Torque transmission, Torsional buckling capacities, Fundamental lateral Natural frequency, Bernoulli Euler theory, Timoshenko beam theory, Static analysis, Modal analysis, Buckling analysis, Ansys.

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Karlskrona, April 2007

Sanjay Gummadi

Akula Jagadeesh Kumar

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1. Notations

A_{ij}	Extensional stiffness matrix, MPa-m
B_{ij}	Coupling stiffness matrix, Mpa-m ²
d_i	Inner diameter of the shaft, mm
d_0	Outer diameter of the shaft, mm
D_{ij}	Bending stiffness matrix, Mpa-m ³
C	Cos θ
E	Young's Modulus, Gpa
E_{11}	Longitudinal elastic modulus of lamina, Gpa
E_{22}	Transverse elastic modulus of lamina, Gpa
E_x	Elastic modulus of the shaft in axial direction(X), GPA
E_y	Elastic modulus of the shaft in tangential direction(Y), Gpa
$E_{x\text{lamina}}$	Elastic modulus of lamina in X- direction, Gpa
$E_{y\text{lamina}}$	Elastic modulus of lamina in Y- direction, Gpa
f_{nbe}	Natural Frequency based on Bernoulli-Euler beam theory,Hz
f_{nt}	Natural Frequency based on Timoshenko beam theory, Hz
G	Shear Modulus, Gpa
G_{12}	Shear modulus of lamina in 12- direction, Gpa
G_{xy}	Shear modulus of the shaft in XY- direction, Gpa
$G_{xy\text{lamina}}$	Shear modulus of the lamina in XY- direction, Gpa
I_x	Moment of inertia of cross-section of the shaft, m ⁴
K_s	Shear coefficient of the lateral natural frequency
L	Length of the shaft, m
m	Weight of the shaft, Kg
m_l	Mass of the shaft per unit length, kg/m
M_x	Bending moment per unit length X-direction, N-m/m

M_y	Bending moment per unit length in Y-direction, N-m/m
M_{xy}	Twisting moment per unit length in XY-direction, N-m/m
n	Total Number of plies
N_{max}	Maximum speed of the shaft, r.p.m
N_{crbe}	Critical Speed of the shaft based on Bernoulli-Euler beam theory, r.p.m
N_x	Normal force per unit length in X-direction, N/m
N_y	Normal force per unit length in Y-direction, N/m
N_{xy}	Shear force per unit length in XY-direction, N/m
r	Mean radius of the shaft, m
S	$\sin\theta$
S_s	Shear Strength, Mpa
S_y	Yield Strength, Mpa
S_1^t	Ultimate longitudinal tensile strength, Mpa
S_1^c	Ultimate longitudinal compressive strength, Mpa
S_2^t	Ultimate transverse tensile strength, Mpa
S_2^c	Ultimate transverse compressive strength, M Pa
S_{12}	Ultimate in-plane shear strength, Mpa
t	Thickness of shaft, mm
T	Torque transmission capacity of the shaft, N-m
T_{cr}	Torsional buckling capacity of the shaft, N-m
ν	Poisson's ratio
ν_{12}	Major Poisson's ratio
ν_{21}	Minor Poisson's ratio
ρ	Density of the shaft material, kg/m^3
θ	Fiber orientation angle, degrees
ϵ_1	Normal strain in longitudinal direction
ϵ_2	Normal strain in transverse direction

γ_{12}	Shear strain in 12- direction
ϵ_x	Normal strain in X- direction
ϵ_y	Normal strain in Y- direction
γ_{xy}	Shear strain in XY- direction
κ_x^0	Midplane curvature in X-direction
κ_y^0	Midplane curvature in Y-direction
κ_{xy}^0	Midplane twisting curvature in X-direction
ϵ_x^0	Midplane extensional strain in X-direction
ϵ_y^0	Midplane extensional strain in X-direction
γ_{xy}^0	Midplane shear strain in XY-direction
σ_1 Mpa	Normal Stress acting in the longitudinal direction of a lamina,
σ_2	Normal Stress acting along the Transverse direction of a lamina, Mpa
σ_x	Normal Stress acting along X- direction of a lamina, Mpa
σ_y	Normal Stress acting along the Y- direction of a lamina, Mpa
τ_{12}	Shear Stress acting in 12-direction of a lamina, Mpa
τ_{xy}	Shear Stress acting in XY- direction of a lamina, Mpa
τ_{cr}	Critical Shear stress, Mpa
ANSYS	Analysis System Software
CLT	Classical Lamination Theory

2. Introduction

The advanced composite materials such as Graphite, Carbon, Kevlar and Glass with suitable resins are widely used because of their high specific strength (strength/density) and high specific modulus (modulus/density). Advanced composite materials seem ideally suited for long, power driver shaft (propeller shaft) applications. Their elastic properties can be tailored to increase the torque they can carry as well as the rotational speed at which they operate. The drive shafts are used in automotive, aircraft and aerospace applications. The automotive industry is exploiting composite material technology for structural components construction in order to obtain the reduction of the weight without decrease in vehicle quality and reliability. It is known that energy conservation is one of the most important objectives in vehicle design and reduction of weight is one of the most effective measures to obtain this result. Actually, there is almost a direct proportionality between the weight of a vehicle and its fuel consumption, particularly in city driving.

2.1 Description of the Problem

Almost all automobiles (at least those which correspond to design with rear wheel drive and front engine installation) have transmission shafts. The weight reduction of the drive shaft can have a certain role in the general weight reduction of the vehicle and is a highly desirable goal, if it can be achieved without increase in cost and decrease in quality and reliability.

It is possible to achieve design of composite drive shaft with less weight to increase the first natural frequency of the shaft and to decrease the bending stresses using various stacking sequences. By doing the same, maximize the torque transmission and torsional buckling capabilities are also maximized.

2.2 Aim and Scope of the Work

This work deals with the replacement of a conventional steel drive shaft with E-Glass/ Epoxy, High Strength Carbon/Epoxy and High Modulus Carbon/Epoxy composite drive shafts for an automobile application.

2.3 Optimum Design Using Genetic Algorithm

The design parameters are to be optimized for E-Glass/ Epoxy, High Strength Carbon/Epoxy and High Modulus Carbon/Epoxy composite drive shafts of an automobile using Genetic Algorithm. The purpose of using Genetic Algorithm is to minimize the weight of the shaft, which is subjected to the constraints such as torque transmission, torsional buckling capacities and fundamental lateral natural frequency.

The design parameters to be optimized are,

- ✓ Ply thickness
- ✓ Number of plies required
- ✓ Stacking sequence of Laminate

2.4 Analysis

1. Modelling of the High Strength Carbon/Epoxy composite drive shaft using ANSYS.
2. Static, Modal and Buckling analysis are to be carried out on the finite element model of the High Strength Carbon/Epoxy composite drive shaft using ANSYS.
3. To investigate
 - a) The stress and strain distributions in E-Glass/ Epoxy, High Strength Carbon/Epoxy and High Modulus Carbon/Epoxy composite drive shafts using classical lamination theory (CLT).
 - b) The effect of centrifugal forces on the torque transmission capacity of the composite drive shafts.
 - c) The effect of transverse shear and rotary inertia on the fundamental lateral natural frequency of the shaft.

3. Background

Composites consist of two or more materials or material phases that are combined to produce a material that has superior properties to those of its individual constituents. The constituents are combined at a macroscopic level and or not soluble in each other. The main difference between composite and an alloy are constituent materials which are insoluble in each other and the individual constituents retain those properties in the case of composites, where as in alloys, constituent materials are soluble in each other and forms a new material which has different properties from their constituents.

3.1 Classification of Composites

Composite materials can be classified as

- Polymer matrix composites
- Metal matrix composites
- Ceramic Matrix

Technologically, the most important composites are those in which the dispersed phase is in the form of a fiber. The design of fiber-reinforced composites is based on the high strength and stiffness on a weight basis. Specific strength is the ratio between strength and density. Specific modulus is the ratio between modulus and density. Fiber length has a great influence on the mechanical characteristics of a material. The fibers can be either long or short. Long continuous fibers are easy to orient and process, while short fibers cannot be controlled fully for proper orientation. Long fibers provide many benefits over short fibers. These include impact resistance, low shrinkage, improved surface finish, and dimensional stability. However, short fibers provide low cost, are easy to work with, and have fast cycle time fabrication procedures. The characteristics of the fiber-reinforced composites depend not only on the properties of the fiber, but also on the degree to which an applied load is transmitted to the fibers by the matrix phase.

The principal fibers in commercial use are various types of glass, carbon, graphite and Kevlar. All these fibers can be incorporated into a matrix either in continuous lengths or in discontinuous lengths as shown in the Fig 3.1. The matrix material may be a plastic or rubber polymer, metal or

ceramic. Laminate is obtained by stacking a number of thin layers of fibers and matrix consolidating them to the desired thickness. Fiber orientation in each layer can be controlled to generate a wide range of physical and mechanical properties for the composite laminate.

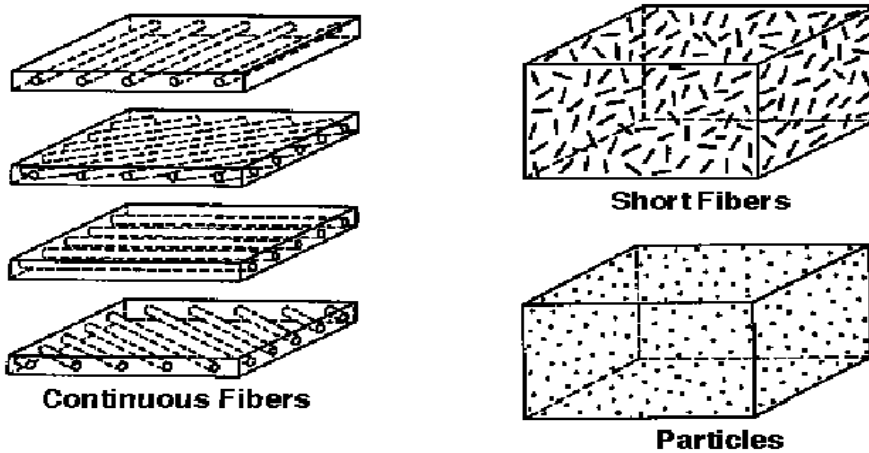


Figure 3.1 Types of fibers

3.2 Advantages of Fiber Reinforced Composites

The advantages of composites over the conventional materials are [1, 2]

- High strength to weight ratio
- High stiffness to weight ratio
- High impact resistance
- Better fatigue resistance
- Improved corrosion resistance
- Good thermal conductivity
- Low Coefficient of thermal expansion. As a result, composite structures may exhibit a better dimensional stability over a wide temperature range.
- High damping capacity.

3.3 Limitations of composites

The limitations of composites are [1, 2],

- Mechanical characterization of a composite structure is more complex than that of a metallic structure
- The design of fiber reinforced structure is difficult compared to a metallic structure, mainly due to the difference in properties in directions
- The fabrication cost of composites is high
- Rework and repairing are difficult
- They do not have a high combination of strength and fracture toughness as compared to metals
- They do not necessarily give higher performance in all properties used for material selection

3.4 Applications of Composites

The common applications of composites are extending day by day. Nowadays they are used in medical applications too. The other fields of applications are,

- **Automotive** : Drive shafts, clutch plates, engine blocks, push rods, frames, Valve guides, automotive racing brakes, filament-wound fuel tanks, fiber Glass/Epoxy leaf springs for heavy trucks and trailers, rocker arm covers, suspension arms and bearings for steering system, bumpers, body panels and doors
- **Aircraft**: Drive shafts, rudders, elevators, bearings, landing gear doors, panels and floorings of airplanes etc.
- **Space**: payload bay doors, remote manipulator arm, high gain antenna, antenna ribs and struts etc.
- **Marine**: Propeller vanes, fans & blowers, gear cases, valves & strainers, condenser shells.
- **Chemical Industries**: Composite vessels for liquid natural gas for alternative fuel vehicle, racked bottles for fire service, mountain climbing, under ground storage tanks, ducts and stacks etc.
- **Electrical & Electronics**: Structures for overhead transmission lines for railways, Power line insulators, Lighting poles, Fiber optics tensile members etc.

- **Sports Goods:** Tennis rackets, Golf club shafts, Fishing rods, Bicycle framework, Hockey sticks, Surfboards, Helmets and others.

3.5 Purpose of the Drive Shaft (or Propeller shaft)

The torque that is produced from the engine and transmission must be transferred to the rear wheels to push the vehicle forward and reverse. The drive shaft must provide a smooth, uninterrupted flow of power to the axles. The drive shaft and differential are used to transfer this torque.

3.6 Functions of the Drive Shaft

- 1 First, it must transmit torque from the transmission to the differential gear box.
- 2 During the operation, it is necessary to transmit maximum low-gear torque developed by the engine.
- 3 The drive shafts must also be capable of rotating at the very fast speeds required by the vehicle.
- 4 The drive shaft must also operate through constantly changing angles between the transmission, the differential and the axles. As the rear wheels roll over bumps in the road, the differential and axles move up and down. This movement changes the angle between the transmission and the differential.
- 5 The length of the drive shaft must also be capable of changing while transmitting torque. Length changes are caused by axle movement due to torque reaction, road deflections, braking loads and so on. A slip joint is used to compensate for this motion. The slip joint is usually made of an internal and external spline. It is located on the front end of the drive shaft and is connected to the transmission.

3.7 Different Types of Shafts

- 1 **Transmission shaft:** These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, overhead shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending moments in addition to

twisting.

- 2 **Machine Shaft:** These shafts form an integral part of the machine itself. For example, the crankshaft is an integral part of I.C. engines slider-crank mechanism.
- 3 **Axle:** A shaft is called “an axle”, if it is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for rotating bodies.
Application: To support hoisting drum, a car wheel or a rope sheave.
- 4 **Spindle:** A shaft is called “a spindle”, if it is a short shaft that imparts motion either to a cutting tool or to a work-piece.
Applications:
 1. Drill press spindles-impart motion to cutting tool (i.e.) drill.
 2. Lathe spindles-impart motion to work-piece.

Apart from, an axle and a spindle, shafts are used at so many places and almost everywhere wherever power transmission is required. Few of them are:

1. **Automobile Drive Shaft:** Transmits power from main gearbox to differential gear box.
2. **Ship Propeller Shaft:** Transmits power from gearbox to propeller attached on it.
3. **Helicopter Tail Rotor Shaft:** Transmits power to tail rotor fan.

This list has no end, since in every machine, gearboxes, automobiles etc. shafts are there to transmit power from one end to other.

3.8 Drive Shaft Arrangement in a Car Model

Conventional two-piece drive shaft arrangement for rear wheel vehicle driving system is shown in figure 3.2 below.

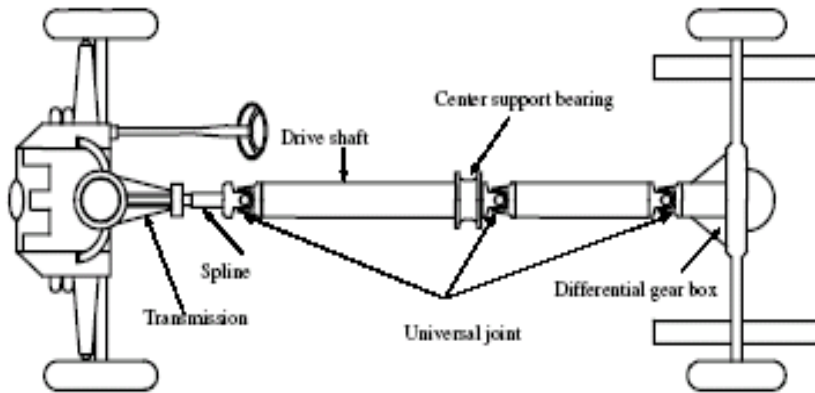


Figure 3.2 Conventional two-piece drive shaft arrangement for rear wheel vehicle driving system

3.9 Part of Drive Shaft and Universal Joint

Parts of drive shaft and universal joint are shown in fig.3.3. Parts of drive shaft and universal joints are

- | | | |
|--------------------------------|------------------------|---------------|
| 1. U-bolt nut | 2. U-bolt washers | 3. U-bolt |
| 4. Universal joint journal | 5. Lubrication fitting | 6. Snap ring. |
| 7. Universal joint sleeve yoke | 8. Spline seal | 9. Dust cap |
| 10. Drive shaft tube | | |

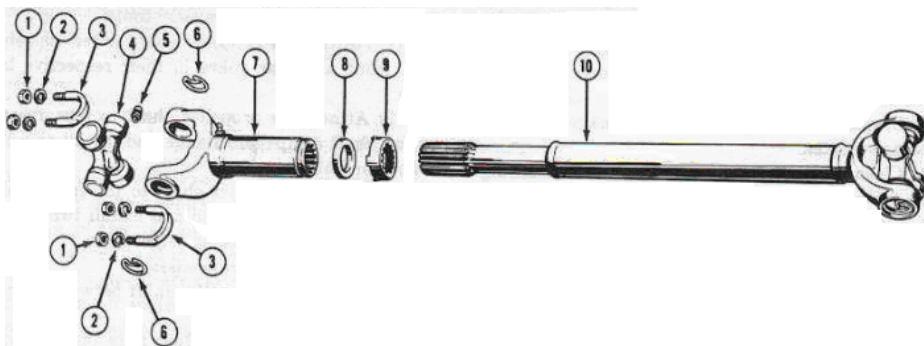


Fig 3.3 Parts of drive shaft and universal joint.

3.10 Demerits of a Conventional Drive Shaft

1. They have less specific modulus and strength [3].
2. Increased weight [3].
3. Conventional steel drive shafts are usually manufactured in two pieces to increase the fundamental bending natural frequency because the bending natural frequency of a shaft is inversely proportional to the square of beam length and proportional to the square root of specific modulus. Therefore the steel drive shaft is made in two sections connected by a support structure, bearings and U-joints and hence over all weight of assembly will be more [4].
4. Its corrosion resistance is less as compared with composite materials [4].
5. Steel drive shafts have less damping capacity

3.11 Merits of Composite Drive Shaft

- 1 They have high specific modulus and strength.
- 2 Reduced weight.
- 3 The fundamental natural frequency of the carbon fiber composite drive shaft can be twice as high as that of steel or aluminium because the carbon fiber composite material has more than 4 times the specific stiffness of steel or aluminium, which makes it possible to manufacture the drive shaft of passenger cars in one piece. A one-piece composite shaft can be manufactured so as to satisfy the vibration requirements. This eliminates all the assembly, connecting the two piece steel shafts and thus minimizes the overall weight, vibrations and the total cost [4].
- 4 Due to the weight reduction, fuel consumption will be reduced [3].
- 5 They have high damping capacity hence they produce less vibration and noise [4].
- 6 They have good corrosion resistance [3].
- 7 Greater torque capacity than steel or aluminium shaft [5].
- 8 Longer fatigue life than steel or aluminium shaft [5].
- 9 Lower rotating weight transmits more of available power [5].

3.12 Drive Shaft Vibration

Vibration is the most common drive shaft problem. Small cars and short vans and trucks (LMV) are able to use a single drive shaft with a slip joint at the front end without experiencing any undue vibration. However, with vehicles of longer wheel base, the longer drive shaft required would tend to sag and under certain operating conditions would tend to whirl and then setup resonant vibrations in the body of the vehicle, which will cause the body to vibrate as the shaft whirls.

Vibration can be either transverse or torsional. Transverse vibration is the result of unbalanced condition acting on the shaft. This condition is usually by dirt or foreign material on the shaft, and it can cause a rather noticeable vibration in the vehicle. Torsional vibration occurs from the power impulses of the engine or from improper universal joint angles. It causes a noticeable sound disturbance and can cause a mechanical shaking. In excess, both types of vibration can cause damage to the universal joints and bearings. Whirling of a rotating shaft happens when the centre of gravity of the shaft mass is eccentric and so is acted upon by a centrifugal force which tends to bend or bow the shaft so that it orbits about the shaft longitudinal axis like a rotating skipping rope. As the speed rises, the eccentric deflection of the shaft increases, with the result that the centrifugal force also will increase. The effect is therefore cumulative and will continue until the whirling become critical, at which point the shaft will vibrate violently.

From the theory of whirling, it has been found that the critical whirling speed of the shaft is inversely proportional to the square of the shaft length. If, therefore, a shaft having, for example, a critical whirling speed of 6000 rev/min is doubled in length, the critical whirling of the new shaft will be reduced to a quarter of this, i.e. the shaft will now begin to rotate at 1500 rev/min. The vibration problem could solve by increasing the diameter of the shaft, but this would increase its strength beyond its torque carrying requirements and at the same time increase its inertia, which would oppose the vehicle's acceleration and deceleration. Another alternative solution frequently adopted by car, van, and commercial vehicle manufacturers is the use of two-piece drive shafts supported by intermediate or centre bearings. But this will increase the cost considerably.

4. Literature Survey

4.1 Composites

The theoretical details of composite materials and composite structures are extensively reviewed [6]. The Spicer U-Joint Division of Dana Corporation for the Ford Econoline van models developed the first composite propeller shaft in 1985. The General Motors pickup trucks, which adopted the Spicer product, enjoyed a demand three times that of projected sales in its first year [4, 5]. John. W. Weeton et al. [7] briefly described the application possibilities of composites in the field of automotive industry to manufacture composite elliptic springs, drive shafts and leaf springs. Beard more and Johnson [8] discussed the potential for composites in structural automotive applications from a structural point of view. Pollard [9] studied the possibility of the polymer Matrix composites usage in driveline applications. Faust et.al, [10] described the considerable interest on the part of both the helicopter and automobile industries in the development of lightweight drive shafts.

Procedure for finding the elastic moduli of anisotropic laminated composites is explained by Azzi.V.D et.al, [11]. Azzi.V.D .et.al, discussed about anisotropic strength of composites [12].

4.2 Torsional Buckling

The problem of general instability under torsional load has been studied by many investigators. Greenhill [13] obtained a solution for the torsional stability of a long shaft. The first analysis of buckling of thin-walled tubes under torsion made by Schwerin [14], but his analysis did not agree with his experimental data. However, all these papers were limited to isotropic materials.

As far as orthotropic materials are concerned, general theories of orthotropic shells were developed by Ambartsumyan [15] and Dong et al. [16]. Cheng and Ho [17] analyzed more generally, the buckling problems of non-homogeneous anisotropic cylindrical shells under combined axial, radial and torsional loads with all four boundary conditions at each end of the cylinder. Lien-Wen Chen et.al. [18] analyzed the stability behaviour of rotating composite shafts under axial compressive loads. A theoretical

analysis was presented for determining the buckling torque of a cylindrical hollow shaft with layers of arbitrarily laminated composite materials by means of various thin-shell theories [19]. Bauchau et al., [20] measured the torsional buckling loads of graphite/epoxy shafts, which were in good agreement with theoretical predictions based on a general shell theory including elastic coupling effects and transverse shearing deformations.

4.3 Lateral Vibrations

Bauchau [21] developed procedure for optimum design of high-speed composite drive shaft made of laminates to increase the first natural frequency of the shaft and to decrease the bending stress. Shell theory based on the critical speed analyses of drive shafts has been presented by Dos Reis et al. [22]. Patricia L.Hetherington [23] investigated the dynamic behaviour of supercritical composite drive shafts for helicopter applications. Ganapathi.et.al [24] extensively studied the nonlinear free flexural vibrations of laminated circular cylindrical shells. A method of analysis involving Love's first approximation theory and Ritz's procedure is used to study the influence of boundary conditions and fiber orientation on the natural frequencies of thin orthotropic laminated cylindrical shells was presented [25].A first order theory was presented by Lee[26] to determine the natural frequencies of an orthotropic shell. Nowinski. J.L. [27] investigated the nonlinear transverse vibrations of elastic orthotropic shells using Von-Karman-Tsien equations.

4.4 Optimization

The optimum design of laminated plates and shells subjected to constraints on strength, stiffness, buckling loads, and fundamental natural frequencies were examined [28]. Methods were proposed for the determination of the optimal ply angle variation through the thickness of symmetric angle-ply shells of uniform thickness [29]. The main features of GAs and several ways in which they can solve difficult design problems were discussed by Gabor Renner et.al.[30]. Raphael T.Haftka [31] discussed extensively about stacking-sequence optimization for buckling of laminated plates by integer programming. The use of a GA to optimize the stacking sequence of

composite laminates for buckling load maximization was studied. Various genetic parameters including the population size, the probability of mutation, and the probability of crossover were optimized by numerical experiments [32]. The use of GAs for the optimal design of symmetric composite laminates subject to various loading and boundary conditions were explained [33]. Kim, et.al. [34] minimized the weight of composite laminates with ply drop under a strength constraint. The working of Simple Genetic Algorithm was explained by Goldberg [35]. Rajeev and Krishnamoorthy [36] proposed a method for converting a constrained optimization problem into an unconstrained optimization problem.

5. Design of Steel Drive Shaft

5.1 Specification of the Problem

The fundamental natural bending frequency for passenger cars, small trucks, and vans of the propeller shaft should be higher than 6,500 rpm to avoid whirling vibration and the torque transmission capability of the drive shaft should be larger than 3,500 Nm. The drive shaft outer diameter should not exceed 100 mm due to space limitations. Here outer diameter of the shaft is taken as 90 mm. The drive shaft of transmission system is to be designed optimally for following specified design requirements as shown in Table 5.1[4].

Table 5.1 Design requirements and specifications

Sl.No	Name	Notation	Unit	Value
1.	Ultimate Torque	T_{\max}	Nm	3500
2.	Max. Speed of shaft	N_{\max}	rpm	6500
3.	Length of shaft	L	mm	1250

Steel (SM45C) used for automotive drive shaft applications. The material properties of the steel (SM45C) are given in Table 5.2 [5]. The steel drive shaft should satisfy three design specifications such as torque transmission capability, buckling torque capability and bending natural frequency.

Table 5.2 Mechanical properties of Steel (SM45C)

Mechanical properties	Symbol	Units	Steel
Young's Modulus	E	GPa	207.0
Shear modulus	G	GPa	80.0
Poisson's ratio	ν	-----	0.3
Density	ρ	Kg/m ³	7600
Yield Strength	S_y	MPa	370
Shear Strength	S_s	MPa	--

5.2 Torque Transmission capacity of the Drive Shaft

$$T = S_s \frac{\pi(d_o^4 - d_i^4)}{16td_o} \quad \text{----- (5.1)}$$

5.3 Torsional Buckling Capacity of the Drive Shaft

$$\text{If } \frac{1}{\sqrt{1-\nu^2}} \frac{L^2 t}{(2r)^3} > 5.5,$$

It is called as Long shaft otherwise it is called as Short & Medium shaft [37].

For long shaft, the critical stress is given by,

$$\tau_{cr} = \frac{E}{3\sqrt{2}(1-\nu^2)^{3/4}} (t/r)^{3/2} \quad \text{----- (5.2)}$$

For short & medium shaft, the critical stress is given by,

$$\tau_{cr} = \frac{4.39E}{(1-\nu^2)} (t/r)^2 \sqrt{1 + 0.0257 (1-\nu^2)^{3/4} \frac{L^3}{(rt)^{1.5}}} \quad \text{----- (5.3)}$$

The relation between the torsional buckling capacity and critical stress is given by,

$$T_{cr} = \tau_{cr} 2\pi r^2 t \quad \text{----- (5.4)}$$

5.4 Lateral or Bending Vibration

The shaft is considered as simply supported beam undergoing transverse vibration or can be idealized as a pinned-pinned beam. Natural frequency can be found using the following two theories.

5.4.1 Bernoulli-Euler Beam Theory-Ncrbe

It neglects the both transverse shear deformation as well as rotary inertia effects. Natural frequency based on the Bernoulli-Euler beam theory is given by [38],

$$f_{nbe} = \frac{\pi p^2}{2L^2} \sqrt{\frac{EI_x}{m_1}} \quad \text{----- (5.5)}$$

Where $p = 1, 2, \dots$

$$N_{crbe} = 60f_{nbe} \quad \text{----- (5.6)}$$

5.4.2 Timoshenko Beam Theory- N_{crit} [39]

It considers both transverse shear deformation as well as rotary inertia effects. Natural frequency based on the Timoshenko beam theory is given by,

$$f_{\text{nt}} = K_s \frac{30\pi p^2}{L^2} \sqrt{\frac{E I^2}{2\rho}} \quad \text{----- (5.7)}$$

$$N_{\text{crit}} = 60f_{\text{nt}} \quad \text{----- (5.8)}$$

$$\frac{1}{K_s^2} = 1 + \frac{n^2 \pi^2 r^2}{2L^2} \left[1 + \frac{f_s E}{G} \right] \quad \text{----- (5.9)}$$

$f_s = 2$ for hollow circular cross-sections

5.4.3 The relation between Timoshenko and Bernoulli-Euler Beam Theories [39]

The relation between Timoshenko and Bernoulli-Euler beam theories is given by,

$$f_{\text{nt}} = K_s f_{\text{nbe}} \quad \text{----- (5.10)}$$

6. Design of a Composite Drive Shaft

6.1 Specification of the Problem

The specifications of the composite drive shaft of an automotive transmission are same as that of the steel drive shaft for optimal design [4].

6.2 Assumptions

1. The shaft rotates at a constant speed about its longitudinal axis.
2. The shaft has a uniform, circular cross section.
3. The shaft is perfectly balanced, i.e., at every cross section, the mass center coincides with the geometric center.
4. All damping and nonlinear effects are excluded.
5. The stress-strain relationship for composite material is linear & elastic; hence, Hooke's law is applicable for composite materials.
6. Acoustical fluid interactions are neglected, i.e., the shaft is assumed to be acting in a vacuum.
7. Since lamina is thin and no out-of-plane loads are applied, it is considered as under the plane stress.

6.3 Selection of Cross-Section

The drive shaft can be solid circular or hollow circular. Here hollow circular cross-section was chosen because:

- The hollow circular shafts are stronger in per kg weight than solid circular.
- The stress distribution in case of solid shaft is zero at the center and maximum at the outer surface while in hollow shaft stress variation is smaller. In solid shafts the material close to the center are not fully utilized.

6.4 Selection of Reinforcement Fiber

Fibers are available with widely differing properties. Review of the design and performance requirements usually dictate the fiber/fibers to be used [1, 2].

- **Carbon/Graphite fibers:** Its advantages include high specific strength and modulus, low coefficient of thermal expansion, and high fatigue strength. Graphite, when used alone has low impact resistance. Its drawbacks include high cost, low impact resistance, and high electrical conductivity.
- **Glass fibers:** Its advantages include its low cost, high strength, high chemical resistance, and good insulating properties. The disadvantages are low elastic modulus, poor adhesion to polymers, low fatigue strength, and high density, which increase shaft size and weight. Also crack detection becomes difficult.
- **Kevlar fibers:** Its advantages are low density, high tensile strength, low cost, and higher impact resistance. The disadvantages are very low compressive strength, marginal shear strength, and high water absorption. Kevlar is not recommended for use in torque carrying application because of its low strength in compression and shear.

Here, both glass and carbon fibers are selected as potential materials for the design of shaft.

6.5 Selection of Resin System

The important considerations in selecting resin are cost, temperature capability, elongation to failure and resistance to impact (a function of modulus of elongation). The resins selected for most of the drive shafts are either epoxies or vinyl esters. Here, epoxy resin was selected due to its high strength, good wetting of fibers, lower curing shrinkage, and better dimensional stability [1, 2]

6.6 Selection of Materials

Based on the advantages discussed earlier, the E-Glass/Epoxy, High Strength Carbon/Epoxy and High Modulus Carbon/Epoxy materials are selected for composite drive shaft. The Table 6.1 shows the properties of the E-Glass/Epoxy, High Strength Carbon/Epoxy and High Modulus Carbon/Epoxy materials used for composite drive shafts [3].

Table 6.1 Properties of E-Glass/Epoxy, HS Carbon/Epoxy and HM Carbon/Epoxy

Sl.No	Property	Units	E-Glass/Epoxy	HS Carbon/Epoxy	HM Carbon/Epoxy
1.	E_{11}	GPa	50.0	134.0	190.0
2.	E_{22}	GPa	12.0	7.0	7.7
3.	G_{12}	GPa	5.6	5.8	4.2
4.	ν_{12}	-	0.3	0.3	0.3
5.	$S_1^t = S_1^c$	MPa	800.0	880.0	870.0
6.	$S_2^t = S_2^c$	MPa	40.0	60.0	54.0
7.	S_{12}	MPa	72.0	97.0	30.0
8.	ρ	Kg/ m^3	2000.0	1600.0	1600.0

6.7 Factor of Safety

The designer must take into account the factor of safety when designing a structure. Since, composites are highly orthotropic and their fractures were not fully studied the factor of safety was taken as 2 [3].

6.8 Torque Transmission Capacity of the Shaft

6.8.1 Stress-Strain Relationship for Unidirectional Lamina

The lamina is thin and if no out-of-plane loads are applied, it is considered as the plane stress problem. Hence, it is possible to reduce the 3-D problem into 2-D problem.

For unidirectional 2-D lamina, the stress-strain relationship is given by,

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \quad \text{----- (6.1)}$$

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12}$$

$$Q_{21} = Q_{12}$$

----- (6.2)

6.8.2 Stress-Strain Relationship for Angle-ply Lamina

The relation between material coordinate system and X-Y-Z coordinate system is shown in Fig 6.1. Coordinates 1, 2, 3 are principal material directions and coordinates X, Y, Z are transformed or laminate axes.

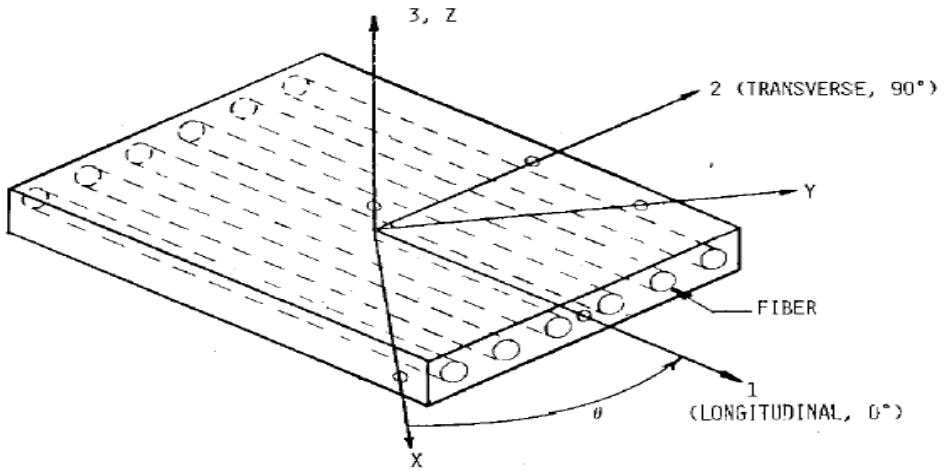


Figure 6.1 Relation between material coordinate system and X-Y coordinate system

For an angle-ply lamina where fibers are oriented at an angle with the positive X-axis (Longitudinal axis of shaft), the effective elastic properties are given by,

$$\frac{1}{E_{x\text{lamina}}} = \frac{1}{E_{11}}C^4 + \left[\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_{11}} \right] S^2C^2 + \frac{1}{E_{22}}S^4 \quad \text{----- (6.3)}$$

$$\frac{1}{E_{y\text{lamina}}} = \frac{1}{E_{11}}S^4 + \left[\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_{11}} \right] S^2C^2 + \frac{1}{E_{22}}C^4 \quad \text{----- (6.4)}$$

$$\frac{1}{G_{xylamina}} = 2 \left[\frac{2}{E_{11}} + \frac{2}{E_{22}} + \frac{2\nu_{12}}{E_{11}} - \frac{1}{G_{12}} \right] S^2C^2 + \frac{1}{G_{12}} [C^4 + S^4] \quad \text{----- (6.5)}$$

The variation of the $E_{x\text{lamina}}$, $E_{y\text{lamina}}$ and $G_{xylamina}$ with ply orientation is shown in Fig 6.2 and 6.3 respectively.

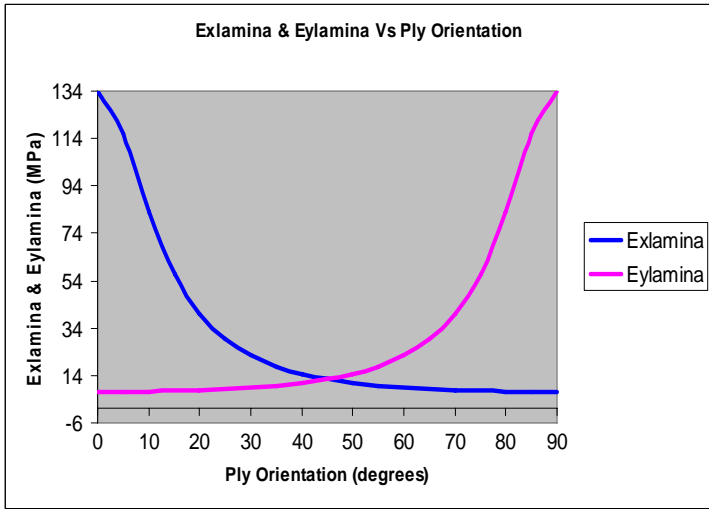


Figure 6.2 The variation of the $E_{xlamina}$ and $E_{ylamina}$ with ply orientation for HS Carbon/Epoxy

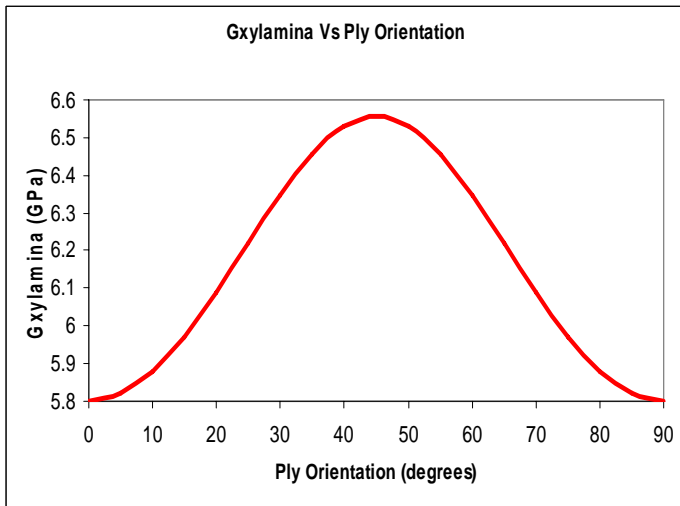


Figure 6.3 The variation of the $G_{xylamina}$ with ply orientation for HS Carbon/Epoxy

The stress strain relationship for an angle-ply lamina is given by,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad \text{---- (6.6)}$$

$$\begin{aligned}
\overline{Q}_{11} &= Q_{11}C^4 + Q_{22}S^4 + 2(Q_{12} + 2Q_{66})S^2C^2 \\
\overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})S^2C^2 + Q_{12}(C^4 + S^4) \\
\overline{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})C^3S - (Q_{22} - Q_{12} - 2Q_{66})CS^3 \\
\overline{Q}_{22} &= Q_{11}S^4 + Q_{22}C^4 + 2(Q_{11} + 2Q_{66})S^2C^2 \\
\overline{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})CS^3 - (Q_{22} - Q_{12} - 2Q_{66})C^3S \\
\overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})S^2C^2 + Q_{66}(S^4 + C^4)
\end{aligned}
\tag{6.7}$$

SHEAR-EXTENSION COUPLING **BENDING-EXTENSION COUPLING**

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}
\tag{6.8}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}
\tag{6.9}$$

BENDING-EXTENSION COUPLING **BEND-TWIST COUPLING**

$$A_{ij} = \sum_{k=1}^n (\overline{Q}_{ij})_k (h_k - h_{k-1}) \quad A_{ij} = \sum_{k=1}^n (\overline{Q}_{ij})_k t_k
\tag{6.10}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^n (\overline{Q}_{ij})_k (h_k^2 - h_{k-1}^2) \quad D_{ij} = \frac{1}{3} \sum_{k=1}^n (\overline{Q}_{ij})_k (h_k^3 - h_{k-1}^3)$$

Where $i, j = 1, 2, 6$.

[A], [B], [D] matrices are called the extensional, coupling, and bending stiffness matrices respectively.

By combining the equations 5.8 and 5.9,

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x^o \\ \kappa_y^o \\ \kappa_{xy}^o \end{Bmatrix} \quad \text{----- (6.11)}$$

For symmetric laminates, the B matrix vanishes and the in plane and bending stiffness are uncoupled. For a symmetric laminate,

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} \quad \text{----- (6.12)}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x^o \\ \kappa_y^o \\ \kappa_{xy}^o \end{Bmatrix} \quad \text{----- (6.13)}$$

$$\begin{Bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \quad \text{----- (6.14)}$$

$$\begin{Bmatrix} \kappa_x^o \\ \kappa_y^o \\ \kappa_{xy}^o \end{Bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{16} \\ d_{12} & d_{22} & d_{26} \\ d_{16} & d_{26} & d_{66} \end{bmatrix} \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} \quad \text{----(6.15)}$$

Where

$$\begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}^{-1} \quad \text{----- (6.16)}$$

$$\begin{bmatrix} d_{11} & d_{12} & d_{16} \\ d_{12} & d_{22} & d_{26} \\ d_{16} & d_{26} & d_{66} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}^{-1} \quad \text{----- (6.17)}$$

$$E_x = \frac{1}{a_{11}} \frac{1}{t} = \text{Young's Modulus of the Shaft in axial direction}$$

$$E_y = \frac{1}{a_{22}} \frac{1}{t} = \text{Young's Modulus of the Shaft in hoop direction}$$

$$G_{xy} = \frac{1}{a_{66}} \frac{1}{t} = \text{Rigidity Modulus of the Shaft in xy plane}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + h \begin{Bmatrix} \kappa_x^o \\ \kappa_y^o \\ \kappa_{xy}^o \end{Bmatrix} \quad \text{----- (6.18)}$$

When a shaft is subjected to torque T, the resultant forces in the laminate by considering the effect of centrifugal forces are

$$N_x = 0 \quad N_y = 2\rho r^2 \omega^2 \quad N_{xy} = \frac{T}{2\pi r^2} \quad \text{----- (6.19)}$$

Stresses in the Kth ply are given by,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k \quad \text{----- (6.20)}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}_k = \begin{bmatrix} C^2 & S^2 & 2CS \\ S^2 & C^2 & -2CS \\ -CS & CS & C^2 - S^2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k \quad \text{----- (6.21)}$$

Knowing the stresses in each ply, the failure of the laminate is determined by using the First Ply Failure criteria. That is, the laminate is assumed to fail when the first ply fails. Here maximum stress theory is used to find the torque transmitting capacity.

6.9 Torsional Buckling Capacity (T_{cr})

Since long thin hollow shafts are vulnerable to torsional buckling, the possibility of the torsional buckling of the composite shaft was checked by the expression for the torsional buckling load T_{cr} of a thin walled orthotropic tube, which was expressed below [40].

$$T_{cr} = (2\pi r^2 t)(0.272)(E_x E_y^3)^{0.25} (t/r)^{1.5} \quad \text{----- (6.22)}$$

This equation has been generated from the equation of isotropic cylindrical shell and has been used for the design of drive shafts. From the equation 5.22, the torsional buckling capability of composite shaft is strongly dependent on the thickness of composite shaft and the average modulus in the hoop direction.

6.10 Lateral or Bending Vibration

The shaft is considered as simply supported beam undergoing transverse vibration or can be idealized as a pinned-pinned beam. Natural frequency can be found using the following two theories.

6.10.1 Bernoulli-Euler Beam Theory-N_{crbe}

It neglects the both transverse shear deformation as well as rotary inertia effects. Natural frequency based on the Bernoulli-Euler beam theory is given by [38],

$$f_{nbe} = \frac{\pi p^2}{2L^2} \sqrt{\frac{E_x I_x}{m_1}} \quad \text{----- (6.23)}$$

Where p=1, 2... ..

$$N_{crbe} = 60f_{nbe} \quad \text{----- (6.24)}$$

6.10.2 Timoshenko Beam Theory- N_{crt}

It considers both transverse shear deformation as well as rotary inertia effects. Natural frequency based on the Timoshenko beam theory is given by [39],

$$f_{nt} = K_s \frac{30\pi p^2}{L^2} \sqrt{\frac{E_x r^2}{2\rho}} \quad \text{----- (6.25)}$$

$$N_{crt} = 60f_{nt} \quad \text{----- (6.26)}$$

Where K_s = shear coefficient of the lateral natural frequency (<1)

$$\frac{1}{K_s^2} = 1 + \frac{n^2 \pi^2 r^2}{2L^2} \left[1 + \frac{f_s E_x}{G_{xy}} \right] \quad \text{----- (6.27)}$$

$f_s = 2$ for hollow circular cross-sections

6.10.3 The relation between Timoshenko and Bernoulli-Euler Beam Theories

The relation between Timoshenko and Bernoulli-Euler beam theories is given by [39],

$$f_{nt} = K_s f_{nbe} \quad \text{----- (6.28)}$$

7. Design Optimization

Optimization of an engineering design is an improvement of a proposed design that results in the best properties for minimum cost. Most of the methods used for design optimization assume that the design variables are continuous. In structural optimization, almost all design variables are discrete. A simple Genetic Algorithm (GA) is used to obtain the optimal number of layers, thickness of ply and fiber orientation of each layer. All the design variables are discrete in nature and easily handled by GA. With reference to the middle plane, symmetrical fiber orientations are adopted.

7.1 How GA differs from the Traditional Optimization Techniques.

GA's differs from traditional optimization algorithm in many ways. A few are listed here [35].

1. GA does not require a problem specific knowledge to carry out a search. GA uses only the values of the objective function. For instance, calculus based search algorithms use derivative information to carry out a search.
2. GA uses a population of points at a time in contrast to the single point approach by the traditional optimization methods. That means at the same time GAs process a number of designs.
3. In GA, the design variables are represented as strings of binary variables that correspond to the chromosomes in natural genetics. Thus the search method is naturally applicable for solving discrete and integer programming problems. For continuous variable, the string length can be varied to achieve any desired resolution.
4. GAs uses randomized operators in place of the usual deterministic ones. In every generation, a new set of strings is produced by using randomized parents selection and crossover from the old generation (old set of strings).

7.2 Objective Function

The objective for the optimum design of the composite drive shaft is the minimization of weight, so the objective function of the problem is given as

Weight of the shaft, $m = \rho AL$

$$m = \rho \frac{\pi}{4} (d_o^2 - d_i^2) L$$

7.3 Design Variables

The design variables of the problem are

- Number of plies
- Thickness of the ply
- Stacking Sequence

The limiting values of the design variables are given as follows,

1. $n \geq 0$
2. $-90 \leq \theta_k \leq 90$
3. $0.1 \leq t_k \leq 0.5$

Where $k = 1, 2 \dots n$ and $n = 1, 2, 3, \dots 32$

The number of plies required depends on the design constraints, allowable material properties, thickness of plies and stacking sequence. Based on the investigations it was found that up to 32 numbers of plies are sufficient.

7.4 Design Constraints

1. Torque transmission capacity of the shaft $T \geq T_{\max}$
2. Bucking torque capacity of the shaft $T_{cr} \geq T_{\max}$
3. Lateral fundamental natural frequency $N \geq N_{\text{crt}}$

➤ The constraint equations may be written as

$$1. C_1 = \begin{cases} \left(1 - \frac{T}{T_{\max}}\right) & \text{If } T < T_{\max} \\ 0 & \text{Otherwise} \end{cases}$$

$$2. C_2 = \begin{cases} \left(1 - \frac{T_{\text{cr}}}{T_{\max}}\right) & \text{If } T_{\text{cr}} < T_{\max} \\ 0 & \text{Otherwise} \end{cases}$$

$$3. C_3 = \begin{cases} \left(1 - \frac{N_{\text{crt}}}{N_{\max}}\right) & \text{If } N_{\text{crt}} < N_{\max} \\ 0 & \text{Otherwise} \end{cases}$$

$$C = C_1 + C_2 + C_3$$

Using the method of Rajeev and Krishnamoorthy [36], the constrained optimization can be converted to unconstrained optimization by modifying the objective function as,

$$\phi = m(1 + k_1 C)$$

For all practical purposes, k_1 is a penalty constant and is assumed to be 10.

7.5 Input GA Parameters

Input GA parameters of E-Glass / Epoxy, HS Carbon/Epoxy and HM Carbon/Epoxy composite drive shafts are shown in the table 7.1.

Table 7.1 Input GA Parameters

Number Of Parameters	$n/2+2$ if n is even
	$(n+1)/2+2$ if n is odd
Total string length	139
Population size	50
Maximum generations	150
Cross-over probability	1
Mutation probability	0.003
String length for number of plies	5
String length for fiber orientation	8
String length for thickness of ply	6

8. Finite Element Analysis

8.1 Introduction

Finite Element Analysis (FEA) is a computer-based numerical technique for calculating the strength and behaviour of engineering structures. It can be used to calculate deflection, stress, vibration, buckling behaviour and many other phenomena. It also can be used to analyze either small or large-scale deflection under loading or applied displacement. It uses a numerical technique called the finite element method (FEM). In finite element method, the actual continuum is represented by the finite elements. These elements are considered to be joined at specified joints called nodes or nodal points. As the actual variation of the field variable (like displacement, temperature and pressure or velocity) inside the continuum is not known, the variation of the field variable inside a finite element is approximated by a simple function. The approximating functions are also called as interpolation models and are defined in terms of field variable at the nodes. When the equilibrium equations for the whole continuum are known, the unknowns will be the nodal values of the field variable.

In this project finite element analysis was carried out using the FEA software ANSYS. The primary unknowns in this structural analysis are displacements and other quantities, such as strains, stresses, and reaction forces, are then derived from the nodal displacements.

8.2 Modelling Linear Layered Shells

SHELL99 may be used for layered applications of a structural shell model as shown in Fig 8.1. SHELL99 allows up to 250 layers. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes.

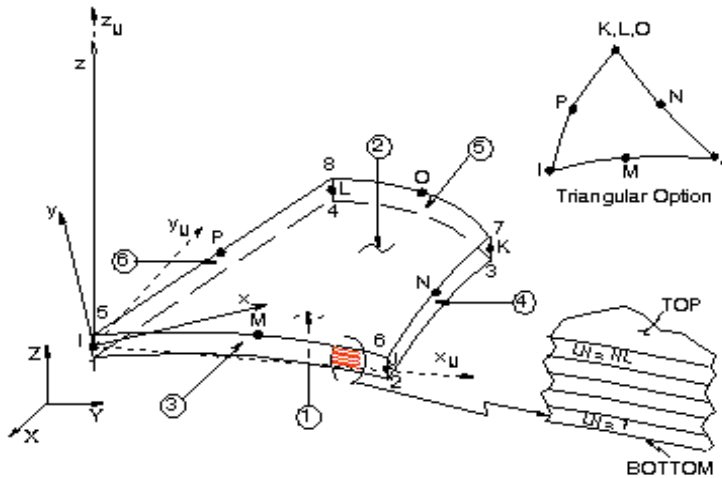


Figure 8.1 SHELL99 Linear Layered Structural Shell

8.2.1 Input Data

The element is defined by eight nodes, average or corner layer thicknesses, layer material direction angles, and orthotropic material properties. A triangular-shaped element may be formed by defining the same node number for nodes K, L and O. The input may be either in matrix form or layer form, depending upon KEYOPT (2). Briefly, the force-strain and moment-curvature relationships defining the matrices for a linear variation of strain through the thickness (KEYOPT (2) = 2) may be defined as:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon \\ \kappa \end{Bmatrix} - \begin{Bmatrix} MT \\ BT \end{Bmatrix}$$

$$[A]_{6 \times 6} = \begin{bmatrix} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 \\ A_2 & A_7 & A_8 & A_9 & A_{10} & A_{11} \\ A_3 & A_8 & A_{12} & A_{13} & A_{14} & A_{15} \\ A_4 & A_9 & A_{13} & A_{16} & A_{17} & A_{18} \\ A_5 & A_{10} & A_{14} & A_{17} & A_{19} & A_{20} \\ A_6 & A_{11} & A_{15} & A_{18} & A_{20} & A_{21} \end{bmatrix} \quad \text{or} \quad [A]_{3 \times 3} = \begin{bmatrix} A_1 & A_2 & A_3 \\ A_2 & A_4 & A_5 \\ A_3 & A_5 & A_6 \end{bmatrix}$$

Sub matrices [B] and [D] are input similarly. Note that all sub matrices are symmetric. {MT} and {BT} are for thermal effects. The layer number (LN) can range from 1 to 250. In this local right-handed system, the x'-axis is rotated an angle THETA (LN) (in degrees) from the element x-axis toward the element y-axis. The total number of layers must be specified (NL). The properties of all layers should be entered (LSYM = 0). If the properties of the layers are symmetrical about the mid-thickness of the element (LSYM = 1), only half of properties of the layers, up to and including the middle layer (if any), need to be entered. While all layers may be printed, two layers may be specifically selected to be output (LP1 and LP2, with LP1 usually less than LP2).

The results of GA forms input to the FEA. Here Finite Element Analysis is done on the HS Carbon/Epoxy drive shaft.

8.3 Static Analysis

Static analysis deals with the conditions of equilibrium of the bodies acted upon by forces. A static analysis can be either linear or non-linear. All types of non-linearities are allowed such as large deformations, plasticity, creep, stress stiffening, contact elements etc. this chapter focuses on static analysis. A static analysis calculates the effects of steady loading conditions on a structure, while ignoring inertia and damping effects, such as those carried by time varying loads. A static analysis is used to determine the displacements, stresses, strains and forces in structures or components caused by loads that do not induce significant inertia and damping effects. A static analysis can however include steady inertia loads such as gravity, spinning and time varying loads.

In static analysis loading and response conditions are assumed, that is the loads and the structure responses are assumed to vary slowly with respect to time. The kinds of loading that can be applied in static analysis includes,

1. Externally applied forces, moments and pressures
2. Steady state inertial forces such as gravity and spinning
3. Imposed non-zero displacements

A static analysis result of structural displacements, stresses and strains and forces in structures for components caused by loads will give a clear idea about whether the structure or components will withstand for the applied maximum forces. If the stress values obtained in this analysis crosses the allowable values it will result in the failure of the structure in the static condition itself. To avoid such a failure, this analysis is necessary.

8.3.1 Boundary Conditions

The finite element model of HS Carbon/Epoxy shaft is shown in Figure 8.2. One end is fixed and torque is applied at other end,

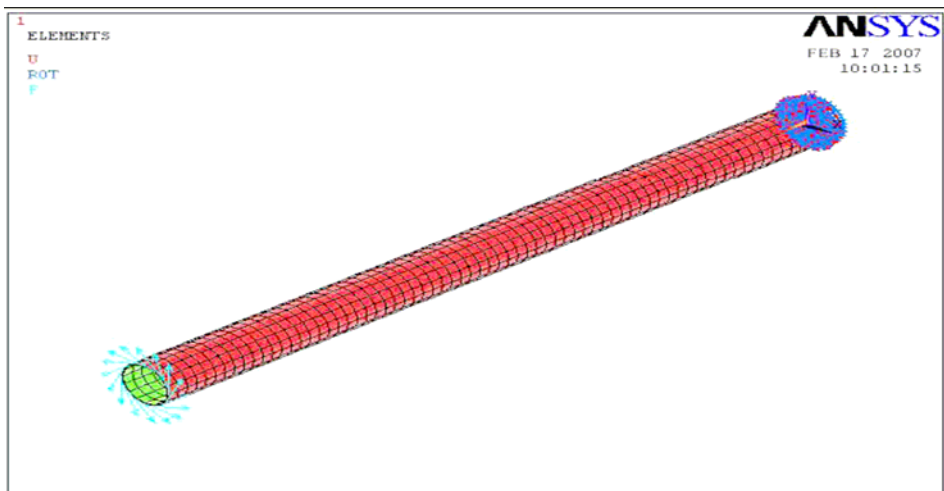


Figure 8.2 Finite element model of HS Carbon/Epoxy shaft

8.4 Modal Analysis

When an elastic system free from external forces is disturbed from its equilibrium position it vibrates under the influence of inherent forces and is said to be in the state of free vibration. It will vibrate at its natural frequency and its amplitude will gradually become smaller with time due to energy being dissipated by motion. The main parameters of interest in free vibration are natural frequency and the amplitude. The natural frequencies

and the mode shapes are important parameters in the design of a structure for dynamic loading conditions.

Modal analysis is used to determine the vibration characteristics such as natural frequencies and mode shapes of a structure or a machine component while it is being designed. It can also be a starting point for another more detailed analysis such as a transient dynamic analysis, a harmonic response analysis or a spectrum analysis. Modal analysis is used to determine the natural frequencies and mode shapes of a structure or a machine component.

The rotational speed is limited by lateral stability considerations. Most designs are sub critical, i.e. rotational speed must be lower than the first natural bending frequency of the shaft. The natural frequency depends on the diameter of the shaft, thickness of the hollow shaft, specific stiffness and the length. Boundary conditions for the modal analysis are shown in Fig 8.3.

8.5 Buckling Analysis

Buckling analysis is a technique used to determine buckling loads (critical loads) at which a structure becomes unstable, and buckled mode shapes (The characteristic shape associated with a structure's buckled response).

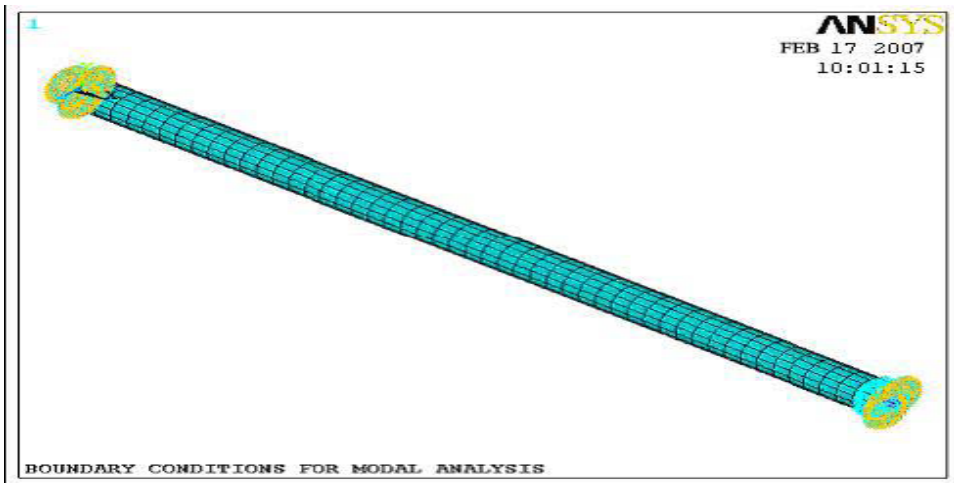


Figure 7.3 Boundary Conditions for the Modal Analysis

For thin walled shafts, the failure mode under an applied torque is torsional buckling rather than material failure. For a realistic driveshaft system, improved lateral stability characteristics must be achieved together with improved torque carrying capabilities. The dominant failure mode, torsional buckling, is strongly dependent on fiber orientation angles and ply stacking sequence.

8.5.1 Types of Buckling Analysis

Two techniques are available in ANSYS for predicting the buckling load and buckling mode shape of a structure. They are,

1. Nonlinear buckling analysis and
2. Eigenvalue (or linear) buckling analysis.

8.5.1.1 Nonlinear Buckling Analysis

Nonlinear buckling analysis is usually the more accurate approach and is therefore recommended for design or evaluation of actual structures. This technique employs a nonlinear static analysis with gradually increasing loads to seek the load level at which your structure becomes unstable.

Using the nonlinear technique, model will include features such as initial imperfections, plastic behavior, gaps, and large-deflection response.

8.5.1.2 Eigenvalue Buckling Analysis

Eigenvalue buckling analysis predicts the theoretical buckling strength (the bifurcation point) of an ideal linear elastic structure. This method corresponds to the textbook approach to elastic buckling analysis: for instance, an Eigenvalue buckling analysis of a column will match the classical Euler solution. However, imperfections and nonlinearities prevent most real-world structures from achieving their theoretical elastic buckling strength. Thus, Eigenvalue-buckling analysis often yields unconservative results, and should generally not be used in actual day-to-day engineering analyses.

9. Results and Discussions

9.1 GA Results

A one-piece composite drive shaft for rear wheel drive automobile was designed optimally by using genetic Algorithm for E-Glass/ Epoxy, High Strength Carbon/Epoxy and High Modulus Carbon/Epoxy composites with the objective of minimization of weight of the shaft which is subjected to the constraints such as torque transmission, torsional buckling capacities and natural bending frequency.

9.1.1 Summarization of GA Results

The GA results are shown in Table 9.1

Table 9.1 GA results

Parameter s	Steel	E-Glass / Epoxy	HS Carbon/Epoxy	HM Carbon/Epoxy
d_o (mm)	90	90	90	90
L (mm)	1250	1250	1250	1250
t_k (mm)	3.318	0.4	0.12	0.12
Optimum no. of Layers	1	17	17	17
t (mm)	3.318	6.8	2.04	2.04
Optimum Stacking sequence	-	[46/64/15/13/39/84/28/20/27] _s	[-56/51/74/82/67/70/13/44/-75] _s	[-65/25/68-63/36/-40/-39/74/-39] _s
Weight (kg)	8.604	4.443	1.1273	1.1274
Weight saving (%)	-	48.36	86.90	86.90

9.1.2 GA Results of E-Glass/Epoxy Drive Shaft

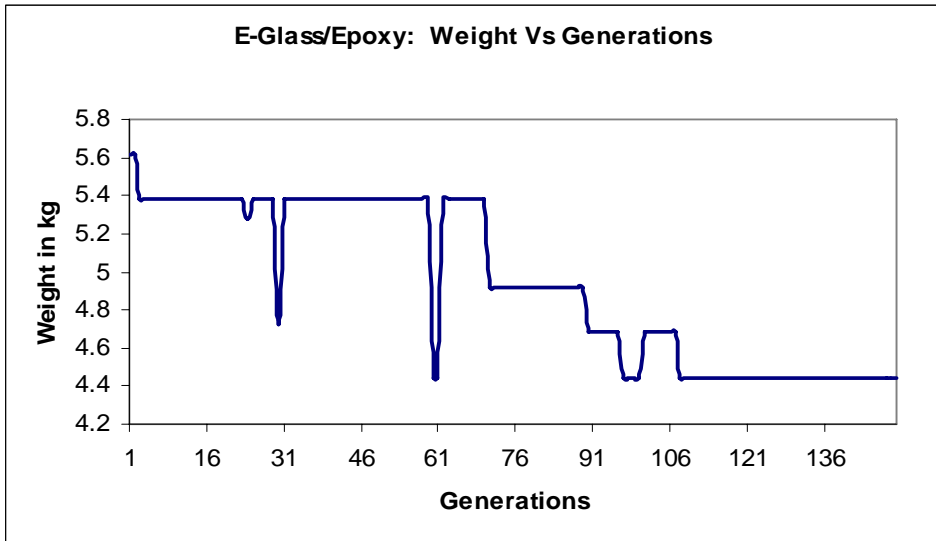


Figure 9.1 Variation of the Weight of E-Glass/Epoxy Drive Shaft with number of generations

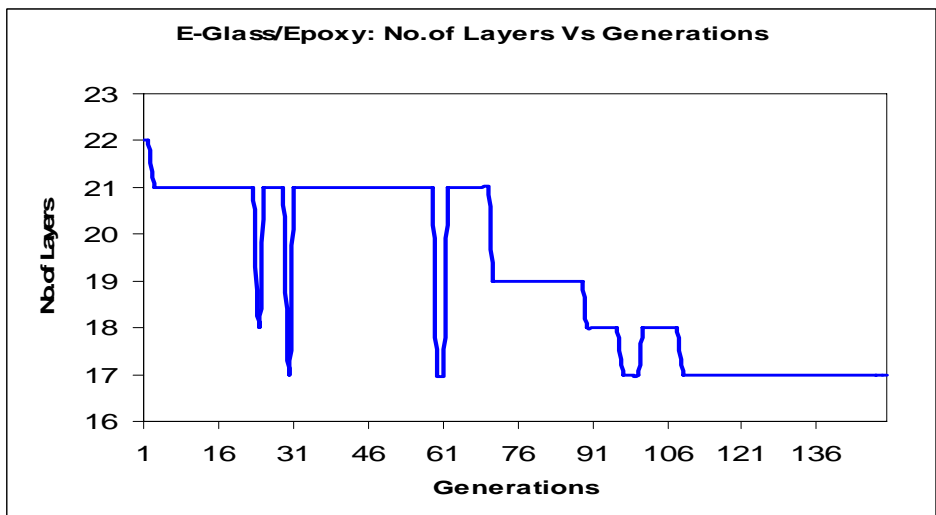


Figure 9.2 Variation of the No. of Layers of E-Glass/Epoxy Drive Shaft with number of generations

9.1.3 GA Results of HS Carbon/Epoxy Drive Shaft

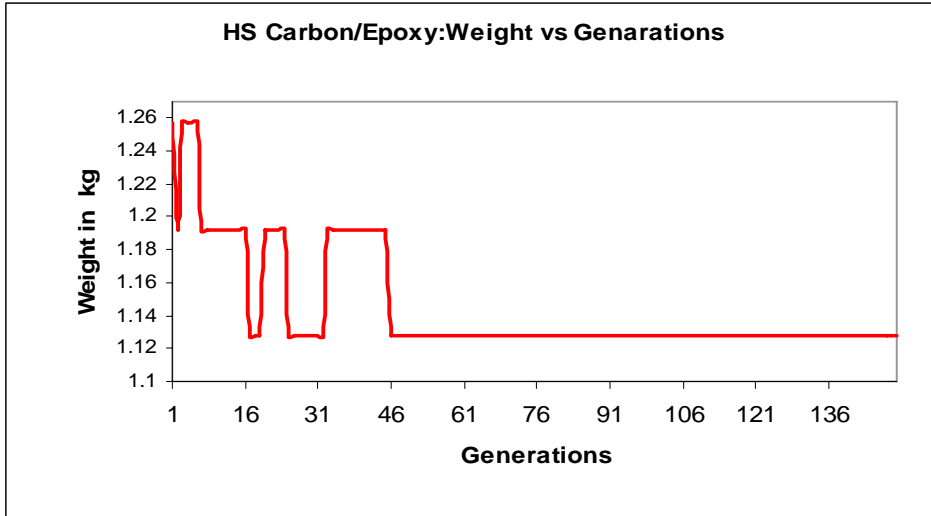


Figure 9.3 Variation of the Weight of HS Carbon/Epoxy Drive Shaft with number of generations

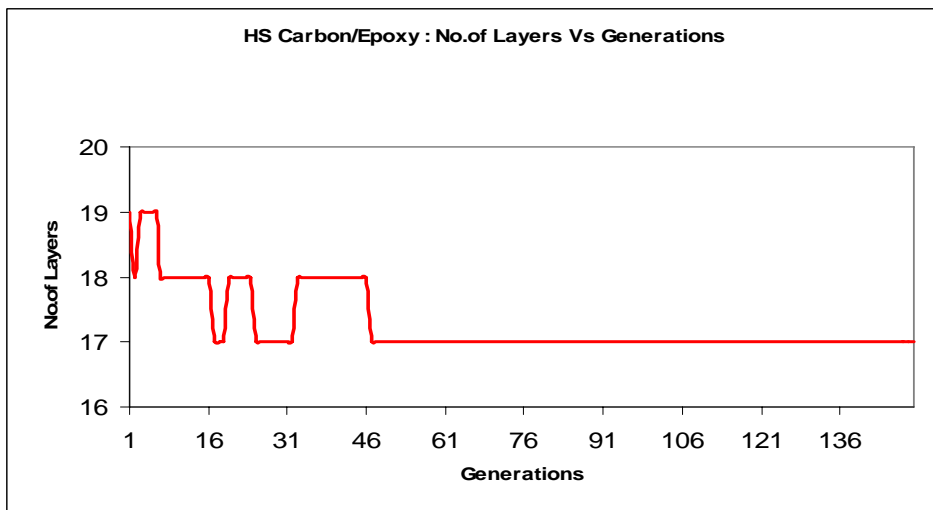


Figure 9.4 Variation of the No. of Layers of HS Carbon/Epoxy Drive Shaft with number of generations

9.1.4 GA Results of HM Carbon/Epoxy Drive Shaft

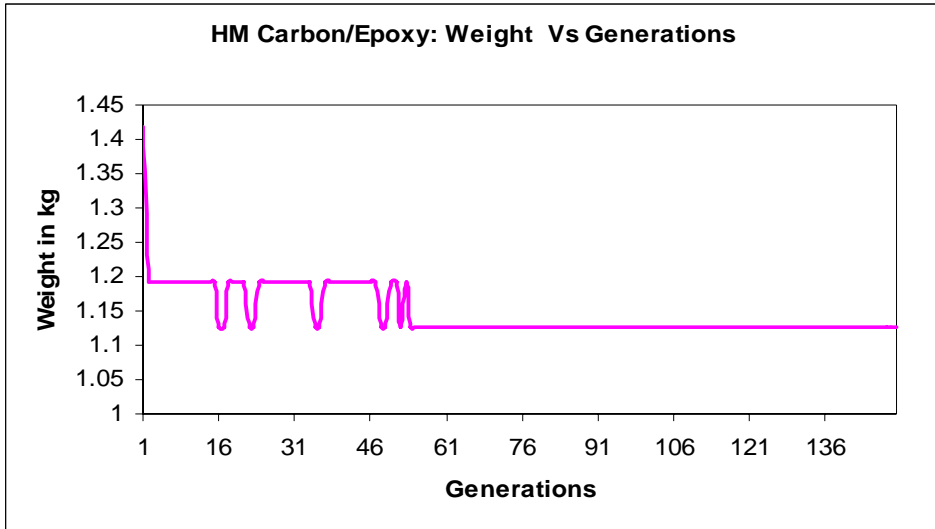


Figure 9.5 Variation of the Weight of HM Carbon/Epoxy Drive Shaft with number of generations

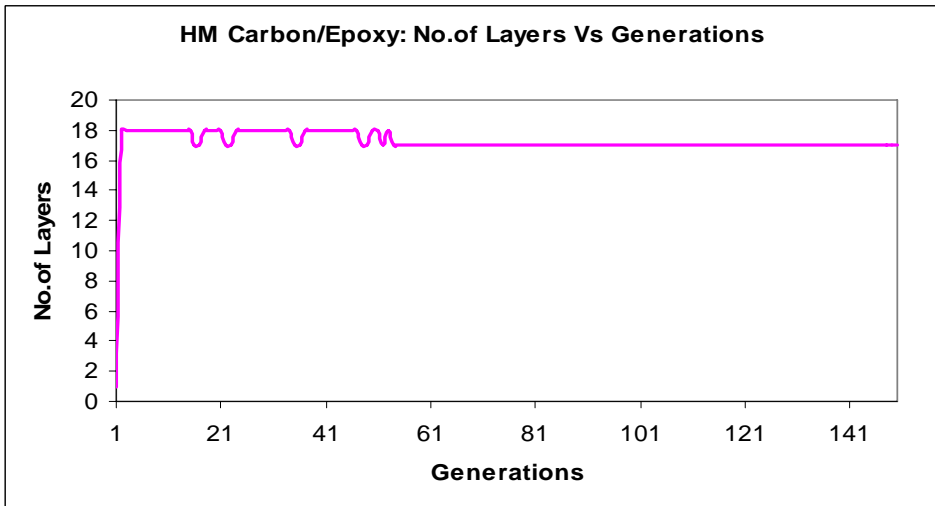


Figure 9.6 Variation of the No. of Layers of HM Carbon/Epoxy Drive Shaft with number of generations

9.2 Stress and Strain Distribution along Thickness of E-Glass/Epoxy Drive Shaft using CLT

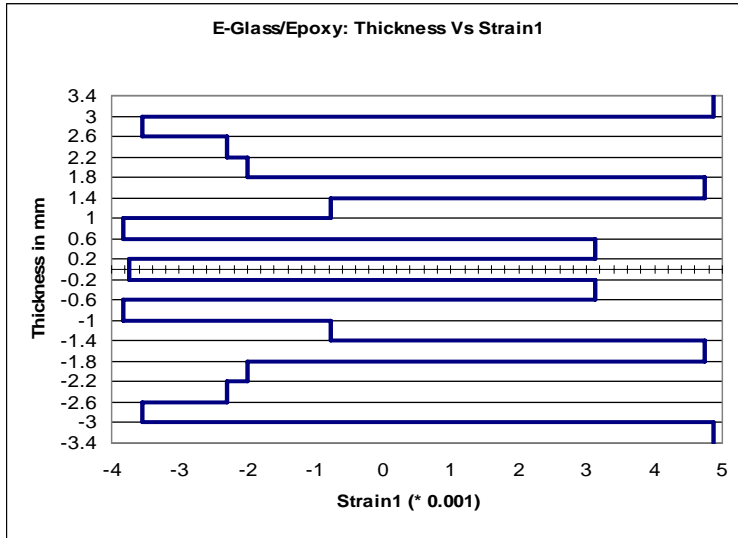


Figure 9.7 Variation of ϵ_1 through thickness of E-Glass/Epoxy Drive Shaft

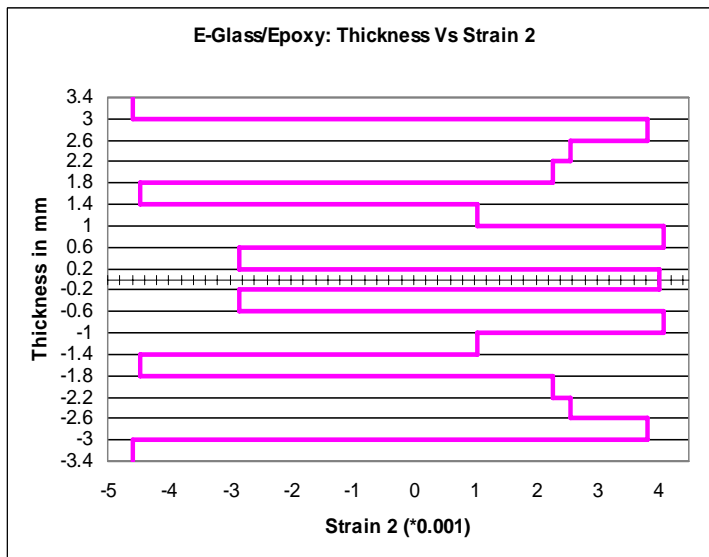


Figure 9.8 Variation of ϵ_2 through thickness of E-Glass/Epoxy Drive Shaft

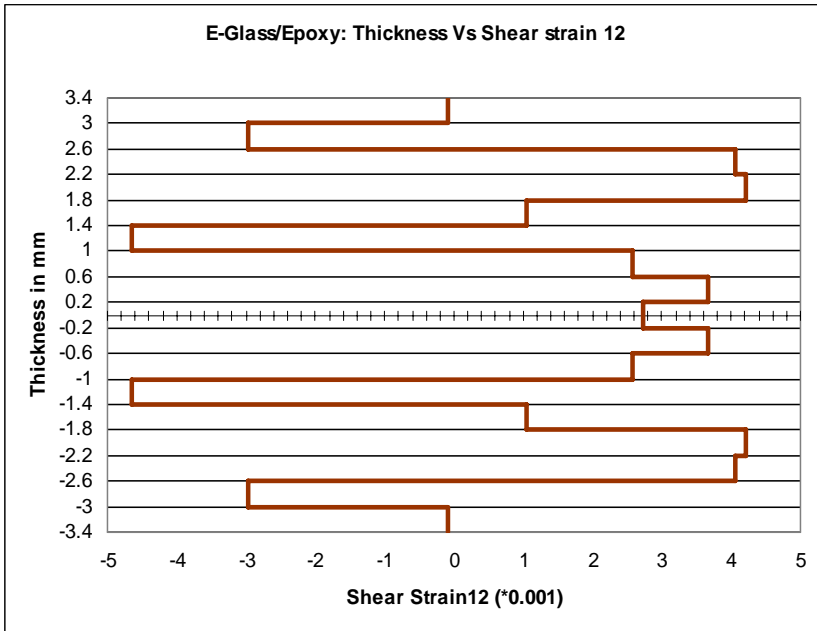


Figure 9.9 Variation of γ_{12} through thickness of E-Glass/Epoxy Drive Shaft

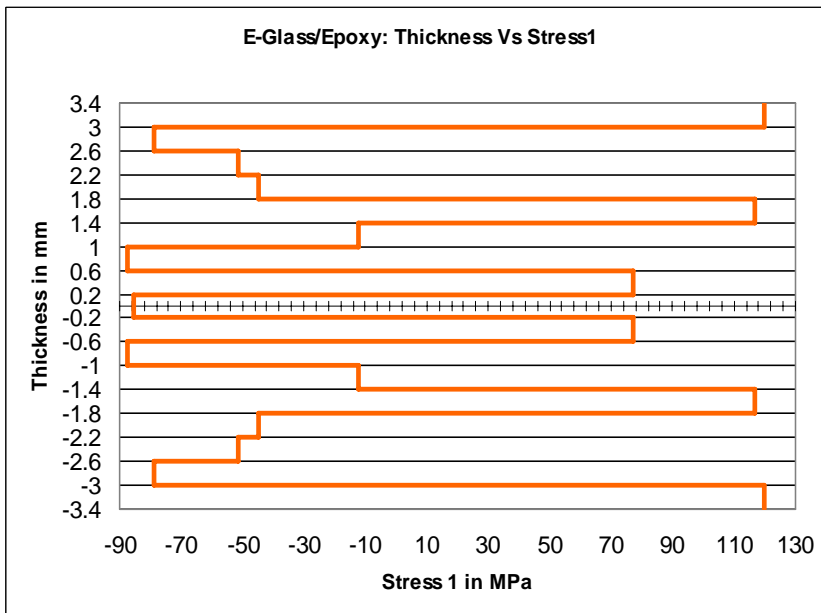


Figure 9.10 Variation of σ_1 through thickness of E-Glass/Epoxy Drive Shaft

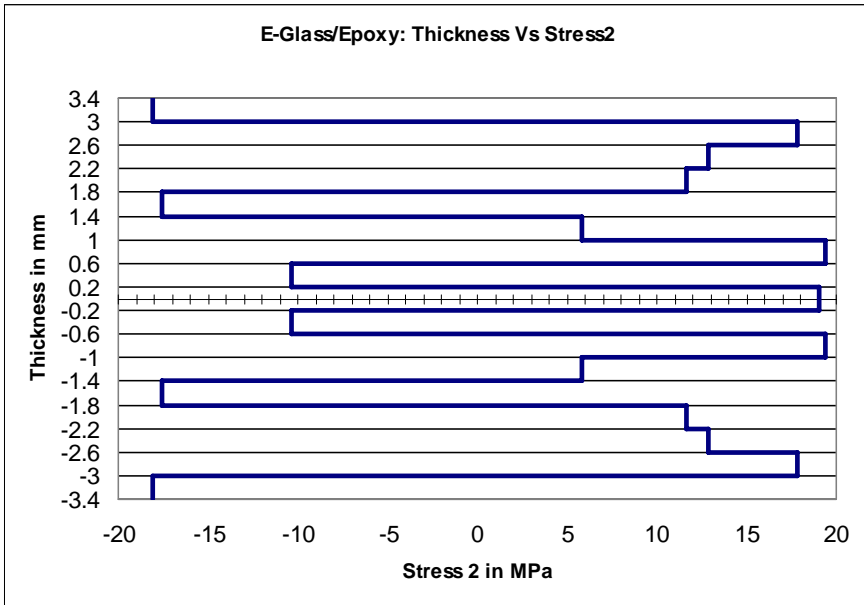


Figure 9.11 Variation of σ_2 through thickness of E-Glass/Epoxy Drive Shaft



Figure 9.12 Variation of τ_{12} through thickness of E-Glass/Epoxy Drive Shaft

From the Figure 9.7-9.12, the following conclusions are drawn and shown in Table 9.2.

Table 9.2 Conclusions from Figure 9.7-9.12

Material	Allowable Stress(MPa)	Predicted Stress (MPa)	Design is OK/NOT
E-Glass/Epoxy	$S_1^t = 400$	119.78	OK
	$S_2^t = 20$	19.38	OK
	$S_{12} = 36$	23.45	OK
	$S_1^c = -400$	-87.34	OK
	$S_2^c = -20$	-18.11	OK
	$S_{12} = -36$	-26.08	OK

9.3 Stress and Strain Distribution along thickness of HS Carbon/Epoxy Drive Shaft using CLT

The stress and strain distribution along thickness of the shaft is shown in figures 9.13-9.18.

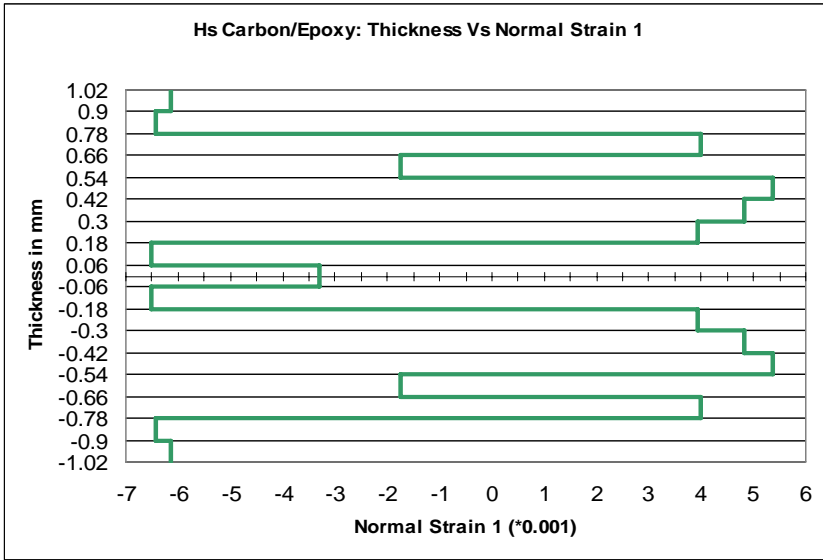


Figure 9.13 Variation of ϵ_1 through thickness of HS Carbon/Epoxy Drive Shaft

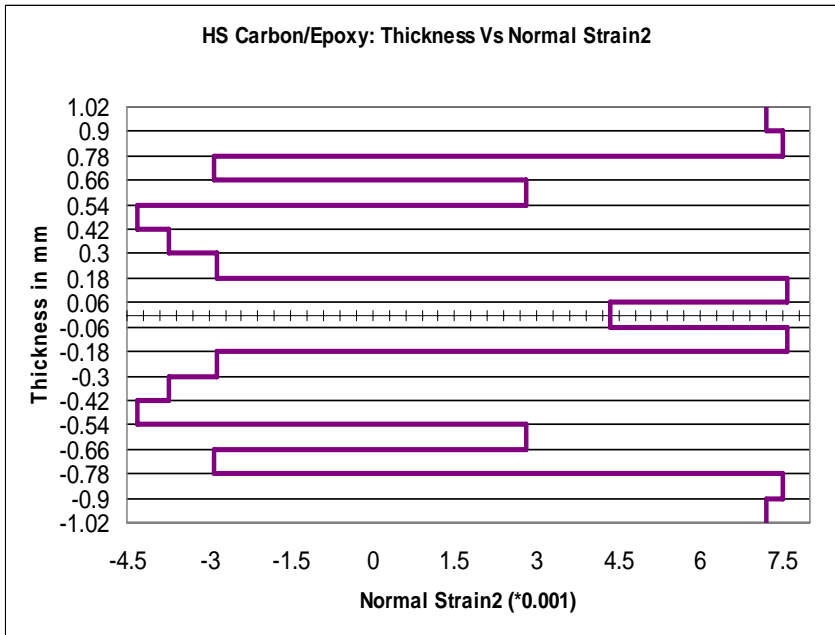


Figure 9.14 Variation of ϵ_2 through thickness of HS Carbon/Epoxy Drive Shaft

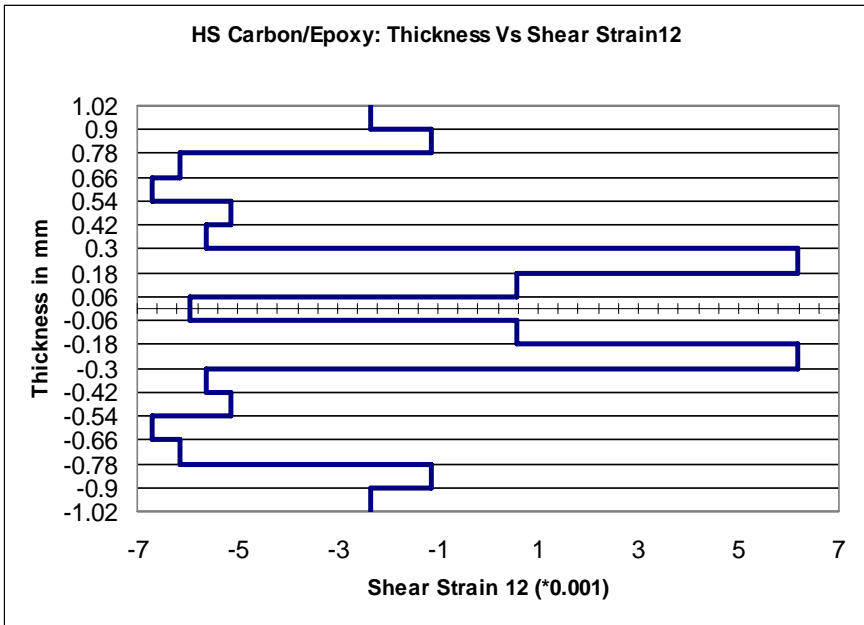


Figure 9.15 Variation of γ_{12} through thickness of HS Carbon/Epoxy Drive Shaft

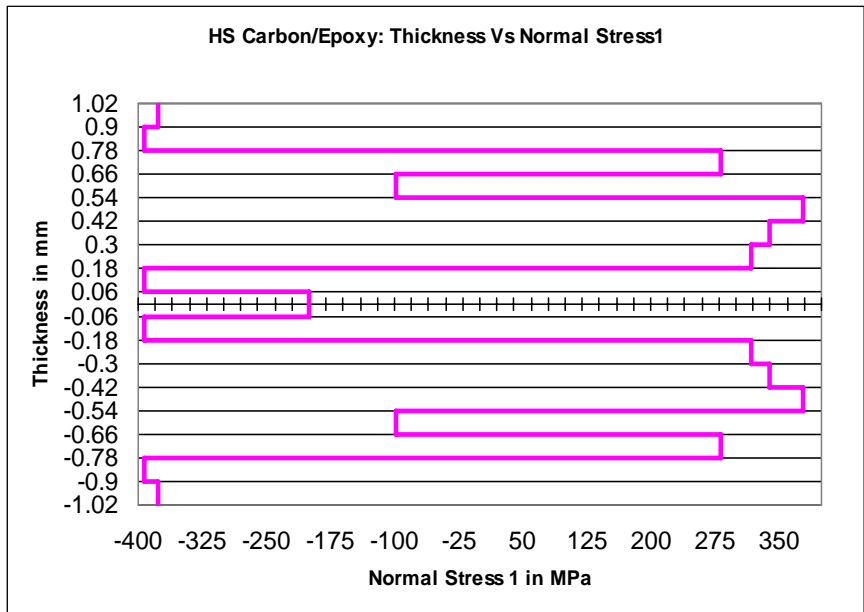


Figure 9.16 Variation of σ_1 through thickness of HS Carbon/Epoxy Drive Shaft

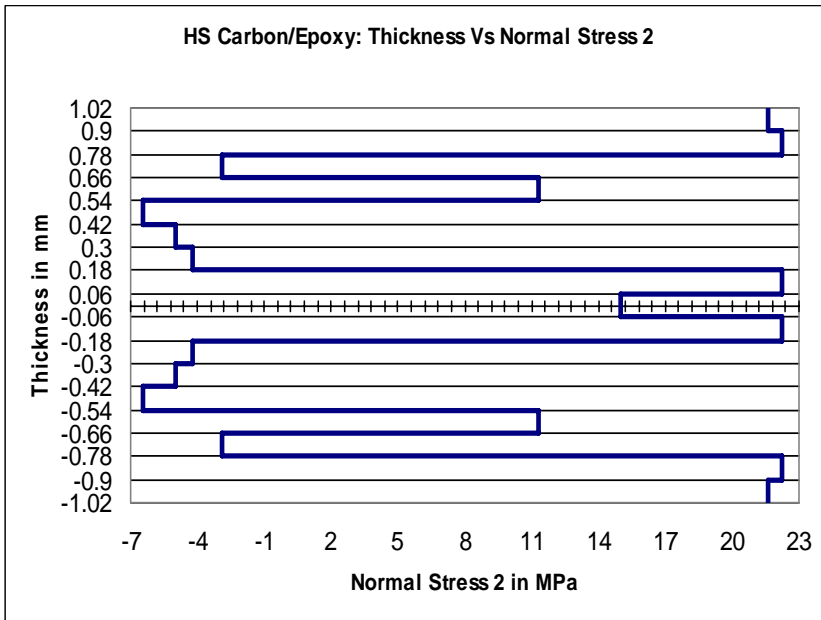


Figure 9.17 Variation of σ_2 through thickness of HS Carbon/Epoxy Drive Shaft

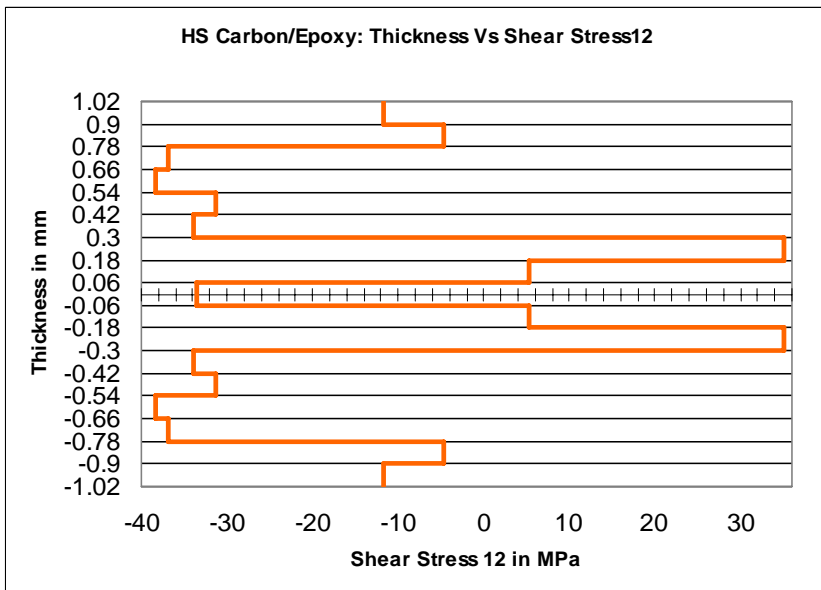


Figure 9.18 Variation of τ_{12} through thickness of HS Carbon/Epoxy Drive Shaft

From Figure 9.13 -9.18, the following conclusions are drawn and shown in Table 9.3.

Table 9.3 Conclusions from Fig 9.13-9.18

Material	Allowable Stress(MPa)	Predicted Stress (MPa)	Design is OK/NOT
HS Carbon/Epoxy	$S_1^t = 440$	378.36	OK
	$S_2^t = 30$	22.24	OK
	$S_{12} = 48.5$	35.06	OK
	$S_1^c = -440$	-392.38	OK
	$S_2^c = -30$	-6.43	OK
	$S_{12} = -48.5$	-38.25	OK

9.4 Stress and Strain distribution along thickness of HM Carbon/Epoxy Drive Shaft using CLT

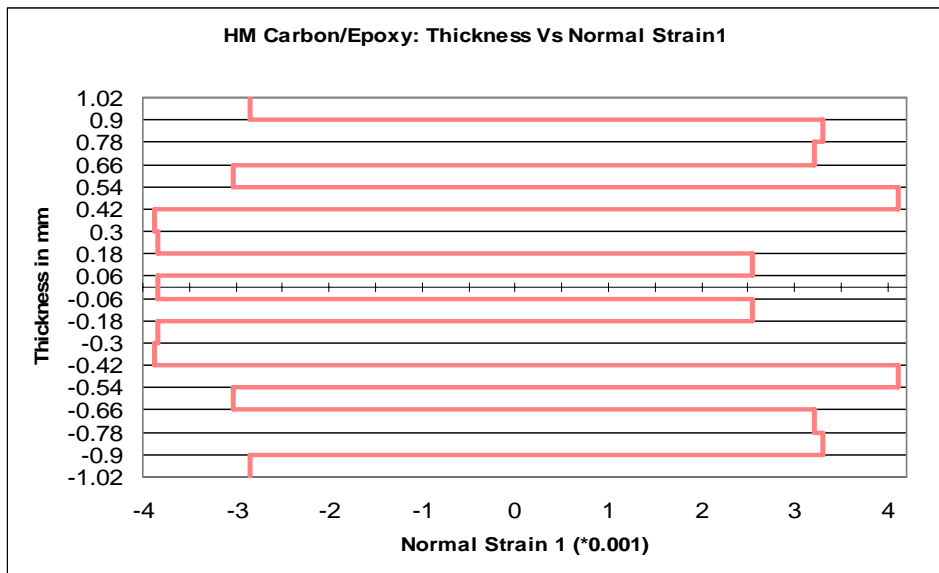


Figure 9.19 Variation of ϵ_1 through thickness of HM Carbon/Epoxy Drive Shaft

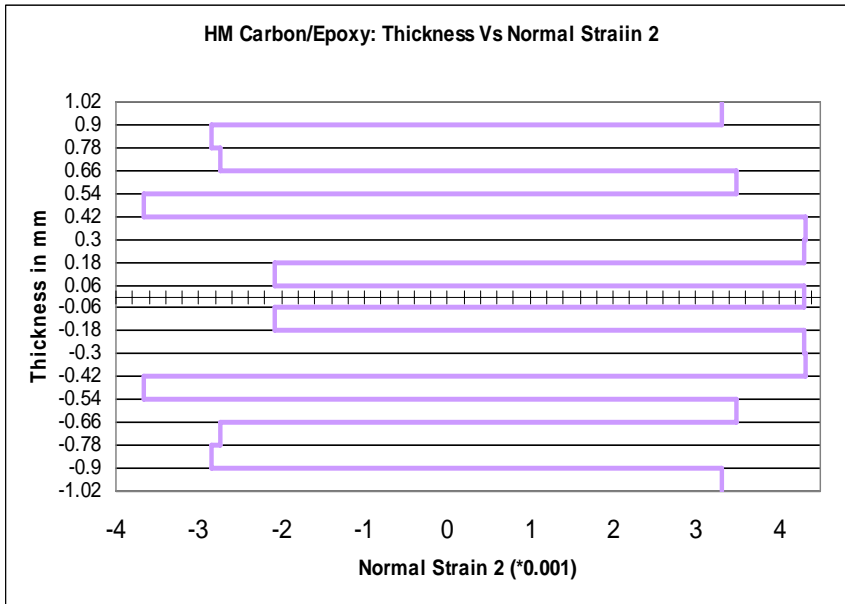


Figure 9.20 Variation of ϵ_2 through thickness of HM Carbon/Epoxy Drive Shaft

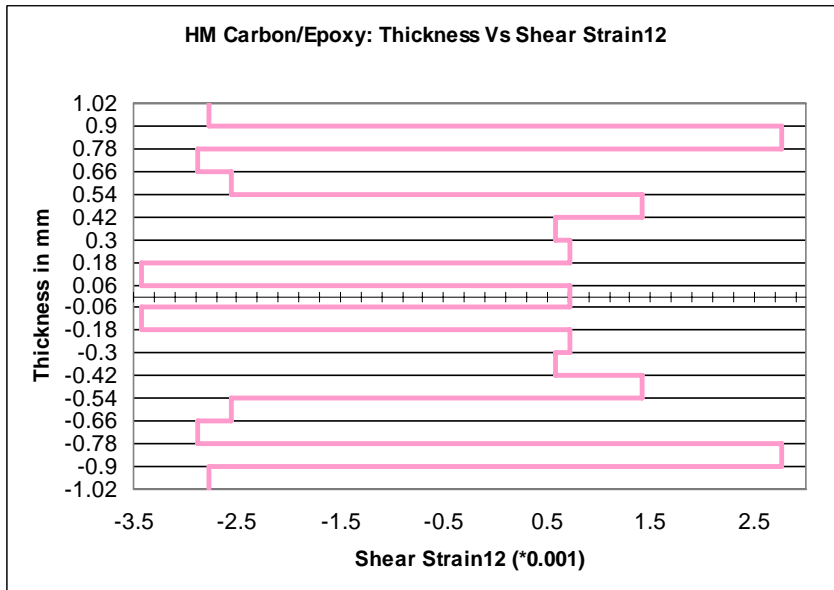


Figure 9.21 Variation of γ_{12} through thickness of HM Carbon/Epoxy Drive Shaft

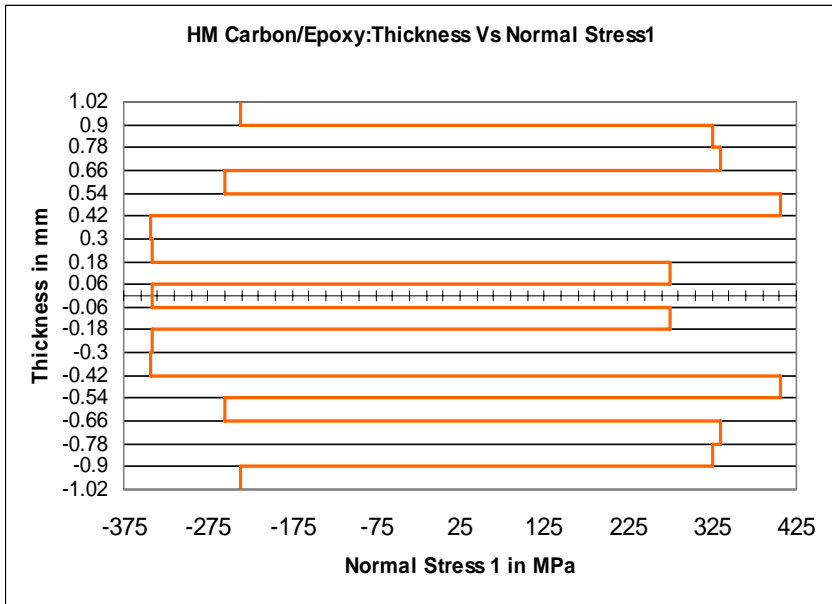


Figure 9.22 Variation of σ_1 through thickness of HM Carbon/Epoxy Drive Shaft

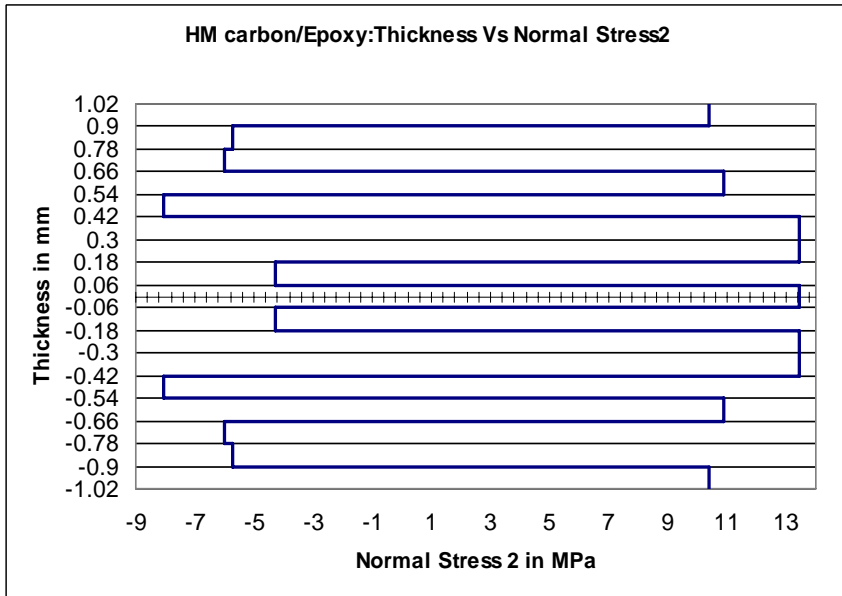


Figure 9.23 Variation of σ_2 through thickness of HM Carbon/Epoxy Drive Shaft

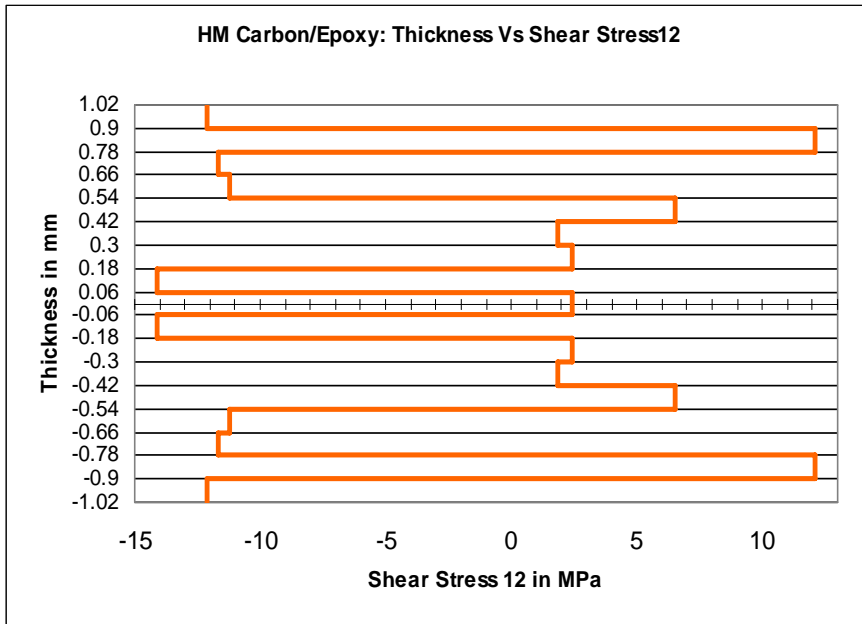


Figure 9.24 Variation of τ_{12} through thickness of HM Carbon/Epoxy Drive Shaft

From the Figure 9.19-9.24, the following conclusions are drawn shown in Table 9.4

Table 9.4 Conclusions from Fig 9.19-9.24

Material	Allowable Stress(MPa)	Predicted Stress (MPa)	Design is OK/NOT
HM Carbon/Epoxy	$S_1^t = 435$	406.54	OK
	$S_2^t = 27$	13.47	OK
	$S_{12} = 15$	12.09	OK
	$S_1^c = -435$	-343.63	OK
	$S_2^c = -27$	-8.07	OK
	$S_{12} = -15$	-14.08	OK

9.5 Deflection

The deflection of E-Glass/Epoxy, HS Carbon/Epoxy and HM Carbon/Epoxy drive shafts are shown in Table 9.5.

Table 9.5 Deflection of Drive Shafts

Material	Deflection(mm)
Steel	0.012407
E-Glass/Epoxy	0.025262
HS Carbon/Epoxy	0.019288
HM Carbon/Epoxy	0.012919

9.6 Elastic Constants of the Composite Drive Shafts

The elastic constants of E-Glass/Epoxy, HS Carbon/Epoxy and HM Carbon/Epoxy drive shafts are shown in Table 9.6.

Table 9.6 Elastic constants of Drive Shafts

Material	E_x (MPa)	E_y (MPa)	G_{xy} (MPa)	ν_{12}
E-Glass/Epoxy	28.99	20.69	9.94	0.35
HS Carbon/Epoxy	26.69	71.68	20.02	0.2
HM Carbon/Epoxy	40.69	62.14	34.20	0.39

9.7 Static Analysis of HS Carbon/Epoxy Drive Shaft

The twist about the axis of the shaft and 1st principal stress along the fiber direction are shown in Fig 9.25 and 9.26 respectively.

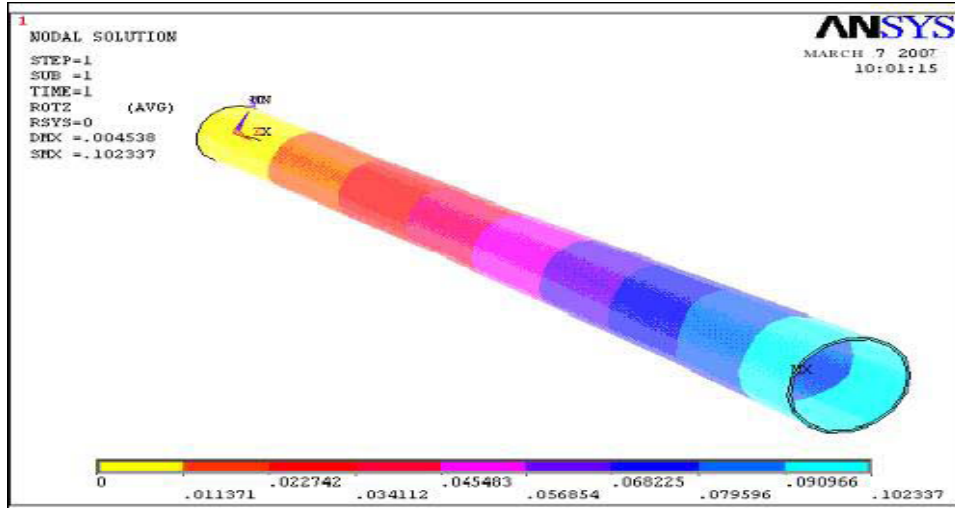


Fig 9.25 Twist about the axis of HS Carbon/Epoxy drive shaft

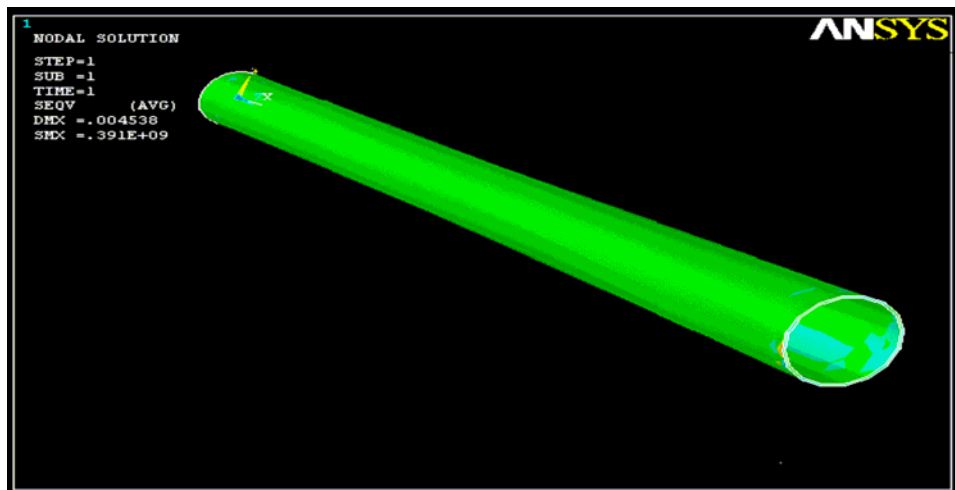


Fig 9.26 1st principal stresses along longitudinal direction for HS Carbon/Epoxy shaft

9.8 Modal Analysis of HS Carbon/Epoxy Drive Shaft

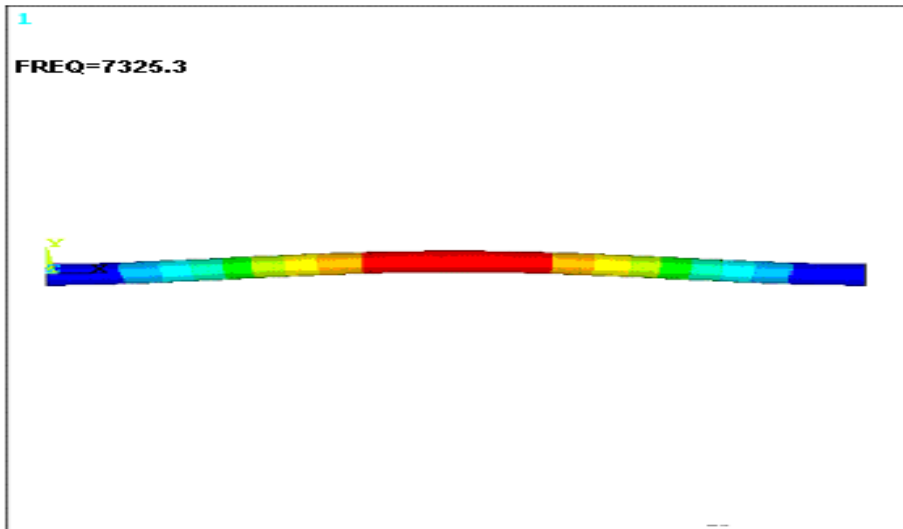


Fig 9.27 1st Vibration Mode shape of HS Carbon/Epoxy shaft

9.9 Buckling Analysis of HS Carbon/Epoxy Drive Shaft

STEP=1
SUB =1
FREQ = 4253.4

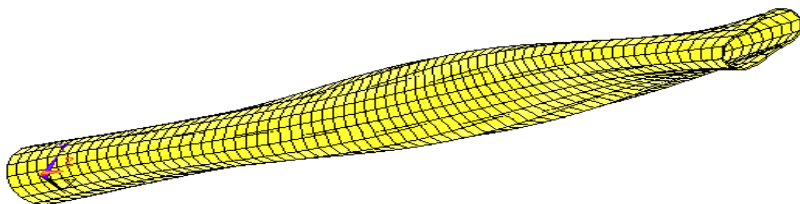


Figure 9.281st Buckling Mode shape of HS Carbon/Epoxy shaft

9.10 The Effect of Centrifugal Forces on the Torque Transmission Capacity

Torque transmission capacities of the drive shafts by considering and neglecting the effect of centrifugal forces are shown in the Table 9.7. It was observed that centrifugal forces will reduce the torque transmission capacity of the shaft.

Table 9.7 Effect of Centrifugal Forces on the Torque Transmission Capacity

Material	E-Glass / Epoxy	HS Carbon/Epoxy	HM Carbon/Epoxy
Optimum Stacking sequence	[46/ -64/ -15/ -13/ 39/ -84/ -28/20/ -27] _s	[-56/ -51/74/ -82/ 67/70/13/ -44/ -75] _s	[-65/25/68/ -63/ 36/ -40/ -39/74/ -39] _s
T (with centrifugal forces) (N-m)	3525.43	3879.26	3656.68
T (with out centrifugal forces) (N-m)	3723.4	4023.5	3875.3

9.11 The Effect of Transverse Shear and Rotary Inertia on the Fundamental Natural Frequency

Natural frequency is calculated by using Bernoulli-Euler and Timoshenko beam theories are shown in Table 8.8. As Bernoulli Euler beam theory neglects the effect of rotary inertia & transverse shear, it gives higher natural frequency.

Table 9.8 Effect of Transverse Shear and Rotary Inertia on the fundamental natural frequency

Material	Steel	E-Glass / Epoxy	HS Carbon/Epoxy	HM Carbon/Epoxy
Optimum Stacking sequence	--	[46/ -64/ -15/ -13/ 39/ -84/ -28/20/ -27]	[-56/ -51/74/ -82/ 67/70/13/ -44/ -75]	[-65/25/68/ -63/ 36/ -40/ -39/74/ -39]
N _{crbe} (rpm)	9662.3 8	6754.91	7663.31	9461.65
N _{crt} (rpm)	9319.9 8	6514.56	7495.42	9270.28

9.12 Torsional Buckling Capacity

The torsional buckling capacity of E-Glass/Epoxy, HS Carbon/Epoxy and HM Carbon/Epoxy drive shafts are shown in Table 9.9.

Table 9.9 Torsional Buckling Capacity of Drive Shafts

Material	Steel	E-Glass / Epoxy	HS Carbon/Epoxy	HM Carbon/Epoxy
Optimum Stacking sequence	--	[46/ -64/ -15/ -13/ 39/ -84/ -28/20/ -27]	[-56/ -51/74/ -82/ 67/70/13/ -44/ -75]	[-65/25/68/ -63/ 36/ -40/ -39/74/ -39]
T _{cr} (Nm)	43857.96	29856.45	3772.11	3765.75

10. Conclusions

The following conclusions are drawn from the present work.

1. The E-Glass/ Epoxy, High Strength Carbon/Epoxy and High Modulus Carbon/Epoxy composite drive shafts have been designed to replace the steel drive shaft of an automobile.
2. A one-piece composite drive shaft for rear wheel drive automobile has been designed optimally by using Genetic Algorithm for E-Glass/ Epoxy, High Strength Carbon/Epoxy and High Modulus Carbon/Epoxy composites with the objective of minimization of weight of the shaft which was subjected to the constraints such as torque transmission, torsional buckling capacities and natural bending frequency.
3. The weight savings of the E-Glass/ Epoxy, High Strength Carbon/Epoxy and High Modulus Carbon/Epoxy shafts were equal to 48.36%, 86.90% and 86.90% of the weight of steel shaft respectively.
4. The optimum stacking sequence of E-Glass/ Epoxy, High Strength Carbon/Epoxy and High Modulus Carbon/Epoxy shafts are shown in Table 9.1,

Table 9.1 Optimum Stacking Sequence

Material	Stacking sequence
E-Glass/Epoxy	$[46/ - 64/ - 15/ - 13/39/ - 84/ - 28/20/ - \overline{27}]_s$
High Strength Carbon/Epoxy	$[-56/ - 51/74/ - 82/67/70/13/ - 44/ - \overline{75}]_s$
High Modulus Carbon/Epoxy	$[-65/25/68/ - 63/36/ - 40/ - 39/74/ - \overline{39}]_s$

5. By using CLT, the variations of the stresses and strains along thickness of the E-Glass/ Epoxy, High Strength Carbon/Epoxy and High Modulus Carbon/Epoxy composite drive shafts were plotted CLT. It has been observed that all the stresses were within the allowable limit.
6. The deflection of Steel, E-Glass/ Epoxy, High Strength Carbon/Epoxy and High Modulus Carbon/Epoxy shafts were equal to 0.012407,

0.025262, 0.019288 and 0.012919 mm respectively.

7. The fundamental natural frequency of Steel, E-Glass/ Epoxy, High Strength Carbon/Epoxy and High Modulus Carbon/Epoxy shafts were 9319.98, 6514.56, 7495.42 and 9270.28 rpm respectively.
8. The torsional buckling capacity of Steel, E-Glass/ Epoxy, High Strength Carbon/Epoxy and High Modulus Carbon/Epoxy shafts were 43857.96, 29856.45, 3772.11 and 3765.75 N-m respectively.
9. The torque transmission capacity of the composite drive shafts has been calculated by neglecting and considering the effect of centrifugal forces and it has been observed that centrifugal forces will reduce the torque transmission capacity of the shaft.
10. Natural frequency using Bernoulli-Euler and Timoshenko beam theories was compared. The frequency calculated by using the Bernoulli Euler beam theory is high, because it neglects the effect of rotary inertia & transverse shear.

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