Unbalanced and Reactive Load Compensation using MMCC-based SATCOMs with Third Harmonic Injection

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Abstract—Modular multilevel cascaded converter-based SATCOMs are scalable to higher voltages without requiring a step-up transformer with multiple windings and can be realised using a low switching frequency, giving lower harmonic content and hence a reduced filtering requirement. The paper presents a new injection technique to extend the operating ranges of MMCC SATCOMs when used for negative sequence and reactive current compensation. A non-sinusoidal voltage or current containing a fundamental and its third harmonic component is injected to achieve phase cluster voltage balance. This technique reduces the maximum dc-link voltage for the star configuration and the maximum converter phase circulating current for the delta case, compared to applying only sinusoidal zero sequence components for mitigating the same degree of load unbalance. The analysis is confirmed experimentally, showing that the third harmonic injection can allow a significant improvement of SATCOM capability for simultaneous compensation of unbalanced load and reactive current.

Index Terms—Delta, Modular Multilevel Cascaded Converter, Star, SATCOM and Unbalanced load.

I. INTRODUCTION

VOLTAGE Source Converter (VSC)-based Static Compensators (STATCOMs) have been applied in power systems for fast dynamic reactive power compensation and voltage control. A STATCOM for high power system may use multiple-winding line frequency transformers to sum the output voltages of several VSCs that have relative phase shifts between them but are not isolated at their DC supply terminals; they may in fact use only a single DC supply [1, 2]. However, the approach now is to synthesize a sinusoidal voltage from several voltage levels, typically obtained from capacitor voltage sources. Three commonly reported multilevel converters, the neutral-point-clamped (NPC), flying capacitor (FC) and cascade H-bridge multilevel converters [2, 3, 4], use capacitor voltages. They obtain more levels, hence higher voltages, by series connecting the devices without device voltage sharing problems. However, for NPC and FC types the number of the achievable voltage levels is often limited to three (or five from \(-v_{dc}\) to \(+v_{dc}\) through zero volt) not only because of voltage unbalance problems but also the voltage clamping requirement. Only the cascaded H-bridge is well suited for high-power applications because of the modular structure that enables higher voltage operation with classic low-voltage semiconductors.

In the mid-1990s a STATCOM based on cascade connection of many H-bridge converter cells with staircase modulation was presented by J Lai & Peng et al [5]. This topology has been further investigated by Marquardt et al [6], though their work was intended for HVDC applications. Since then Modular Multi-level Cascaded Converters (MMCC) have received considerable interest for STATCOM and other applications [7-12]. The modular concept offered by an MMCC makes it easily scalable to higher voltages without requiring a step-up transformer [13]. Also, it realizes a more complex output waveform using a low switching frequency, giving lower harmonic content and hence a reduced filtering requirement. Moreover, the topology is more fault tolerant, with easier manufacturing and maintenance, lower cost, higher reliability and better efficiency. The commonly accepted topology for MMCC sub-modules uses the single-phase H-bridge inverter as the basic building block [13]; a three-level flying capacitor full-bridge inverter has also been used [10]. The three phase limbs of an MMCC may be in either star or delta connection, and the whole may be classified as a Single Star Bridge Converter MMCC (SSBC-MMCC) or Single Delta Bridge Converter (SDBC-MMCC). Both have been researched for use as a STATCOM for voltage control and unbalanced load compensation [13-16].

Load unbalance is common in three-phase systems, usually due to uneven distribution of single-phase loads. Typical examples are lighting loads, single-phase traction, variable speed motor drives etc, which are all subject to large current fluctuations. Switch mode power supplies using single-phase diode rectifiers also introduce harmonic and unbalance currents. Many small renewable energy sourced generators feeding power to the distribution system exacerbates line currents imbalance. STATCOMs are effective and flexible in compensating time-varying unbalanced load currents, and examples of MMCC-based STATCOMs for this type of applications have been reported [17-19]. These are able to supply load demanded negative sequence current directly, so that the grid supplied current, and hence also the voltage at the point of common coupling, can be kept well balanced.

One of the main challenges for using the MMCC-based STATCOM is eliminating the dc-link voltage imbalance, which is caused by the stacked sub-modules in each phase having their module capacitors isolated from each other. This
produces two types of voltage imbalance: intra-phase and inter-phase imbalances. The former denotes individual module capacitor voltages in a phase cluster drifting away from their nominal levels due to unequal charging and discharging. This has been dealt with by applying suitable multilevel PWM schemes combined with closed-loop averaging dc voltage control [20, 21]. The latter, the inter-phase voltage imbalance, denotes differences in converter phase cluster voltages. This is caused by the MMCC-STATCOM supplying unbalanced power to the grid, since the isolated module capacitors prevent active power exchange between phases. Consequently dc-link voltages may drift away from their rated levels, resulting in STATCOM malfunction leading to distorted currents injected into the grid, and over stressing or even damaging the devices. The approach used for an SSSC-MMCC is to inject a sinusoidal zero-sequence voltage in the converter neutral point [17, 22-24]. For SDBC-MMCC a zero-sequence current is applied to circulate in the delta-configured three-phase limbs [19, 25]. However, such a scheme causes serious problems; in the former the injected zero-sequence voltage can cause the converter phase voltages to exceed their rated level, resulting in the SSSC operating in over-modulation mode or even becoming uncontrollable. In the SDBC-MMCC, it can lead to current being too high and exceeding the rated limit. A detailed analysis of such limitations for both the SSSC and SDBC MMCCs has been given in [25, 26], though this was for application in PV power generation. Another publication [19] highlighted the compensation limitations for a star connection STATCOM under unbalanced current compensation, and delta connection under unbalanced voltage compensation, i.e., the singularity issue, without analysing the delta case for load unbalance condition. However, there is no work quantifying clearly the level of load current unbalance in relation to the phase limb voltage rating in star connection, nor to the phase current rating in delta connection. Furthermore, it is necessary to explore new method to mitigate effectively such limitations, and hence extending the MMCC-STATCOM operating ranges under unbalanced load compensation. Suggestions have been made in the context of MMCC grid-connected PV systems [25, 27-29], however the effectiveness of this approach has not been investigated, nor have the limitations been stated, particularly for MMCC-based STATCOMs.

This paper proposes a new control scheme for MMCC-based STATCOMs for unbalanced load current compensation in a power system. The method injects a non-sinusoidal voltage to a star-connected MMCC converter and a non-sinusoidal current to a delta-connected MMCC, to extend the capacitor utilization range. The injected waveform is composed of a fundamental plus its third-harmonic component. Injection of third-order harmonics in PWM controlled converters is a known technique, mostly applied to the modulation reference signals for making full use of the dc-link voltage [30, 31]. A very recent paper described work using third order injection for a grid-connected MMC-HVDC [32] and addressed the same issue as in [30, 31]; it proposed a novel curve-fitting method for evaluating the optimal magnitude and phase of the third harmonic, the objective being to minimize the modulation waveform peak values, and hence increase the capacitor voltage utilization. The work described in this current paper deals with the distinct problem of compensating the unbalanced load current. This relies on zero sequence voltage (for a star connected MMCC) and current (for a delta connected MMCC) to bring about phase limb voltage balance. The third-order injection here is intended to suppress the peak value of the zero sequence components, rather than directly dealing with the AC terminal modulation waveform. The magnitude and phase of the third-harmonic element in this application depend on the zero-sequence element which is evaluated by the three-phase power balance principle. This scheme enables effective power sharing, hence capacitor voltage balance, between phase clusters. Compared to the traditional sinusoidal zero sequence waveform injection, the technique reduces the peak phase voltages and so extends the converter operating range of an MMCC-STATCOM for unbalanced load compensation. The paper analyses and quantifies the reductions of peak phase voltage under different levels of load current imbalance and the results are compared with the conventional purely sinusoidal injection. Experimental results, obtained from a 140V, 1.5kVA, 2-module per phase MMCC, will be discussed and shown to validate the enhanced capabilities offered by the new control scheme.

II. CIRCUIT CONFIGURATION OF MMCC-BASED STATCOMS

An MMCC-based STATCOM may be in either star (SSBC) or delta connections (SDBC) as illustrated in Figs. 1(a) and (b) respectively. For SSBC the neutral points of the supply and converter sides are not connected together. The basic cells in the phase clusters can be either two level H-bridge (2L-HB) or three level flying capacitor converter (3L-FCC). While 2L-HB generates three distinct voltage levels (0, ±V_{dc}), and 3L-FCC generates five levels (0, ±V_{dc}, ±2V_{dc}) respectively. Both topologies, shown in the figure below, are well-known and their applications in STATCOM devices have been reported [7, 10].

Either of these two MMCCs can be connected to the grid at the point of common coupling (PCC) for compensating the reactive power as well as negative sequence current due to unbalanced load current. However as the cells in each phase chain are isolated, the cell voltage may drift away from their rated levels when unbalanced power flows between phases, causing them giving poor performance or even malfunction. The method of using zero sequence voltage/current has been proposed for preventing such problem occurring [23], thus for SSBC-MMCC a zero sequence voltage is injected to cancel out the unbalanced power between phases and for SDBC-MMCC a zero sequence current is added. However the superimposed zero sequence elements result in the rise of SSBC-MMCC phase voltage to exceed its rated level, and current through SDBC-MMCC increases over its limit. Consequently their abilities in compensating unbalanced load current are significantly compromised.
III. PHASE CLUSTER DC-VOLTAGE CAPACITOR BALANCING WITH ZERO SEQUENCE AND THIRD HARMONIC INJECTION

Adding third harmonic voltage components into the zero-sequence voltage in a star connected MMCC-STATCOM, or third harmonic current in its delta connected counterpart, can alleviate the problems of peak voltages or currents exceeding their limits, and so extend the phase power balancing capability in either case. A similar approach has been applied effectively in three-phase voltage source inverters for motor drive applications to extend the inverter dc-link voltage utilization [33]. The approach is analysed as follows, and its effectiveness quantified for both SSBC and SDBC cases.

A. SSBC-MMCC with Third Harmonic Voltage Injection

In this case, the voltage term injected into the phase clusters for balancing cluster powers is the sum of three: zero sequence voltage which is the fundamental element, its third harmonic component and the third harmonic of the PCC voltage expressed as:

\[
v_{n3} = \frac{V_o}{v_o} \sin(\omega t + \phi_o) + \frac{V_0}{6} \sin(3\omega t + 3\phi_o) + \frac{V_p}{6} \sin(3\omega t + 3\phi_{vp})
\]

The addition of 1/6 of the third harmonic components \(v_{n3}\) and \(v_{vp3}\) reduces the peak cluster voltage reference to \(\sqrt{3}/2\) allowing 15% increase in modulation index. Both third harmonic components are added up to the fundamental sequence component to act as the common mode component in all three clusters [25, 27].

Evaluation of all terms on the right hand side (RHS) of (1) relies on accurate estimation of the fundamental zero sequence voltage \(v_o\), which is derived from analysing unbalanced power due to negative sequence voltage and current are described below.

1) Power Imbalance Analysis: When MMCC-STATCOM compensating unbalanced load current, the phase voltages at the PCC and the currents from the MMCC to the grid are given respectively as follows:

\[
v_{sn} = V_p \sin(\omega t + \phi_{vp} - \frac{2\pi}{3}) + V_n \sin(-\omega t + \phi_{vn} - \frac{2\pi}{3}).
\]

and

\[
i_m = I_p \sin(\omega t + \phi_{ip} - \frac{2\pi}{3}) + I_n \sin(-\omega t + \phi_{in} - \frac{2\pi}{3}).
\]

where \(k = 0, 1, 2\) for \(m = a, b, c\) respectively. \(V_p\), and \(V_n\) denote positive and negative sequence voltage magnitudes, likewise \(I_p\), and \(I_n\) for currents.

The phase power obtained by multiplying (2) and (3) contains unbalanced active powers due to the negative sequence elements in both equations, these flow through the converter phases, causing, consequently, the converter inter-cluster dc capacitor voltage imbalance. Injecting zero sequence voltage into phase cluster for canceling this power leads to converter phase voltages become:

\[
v_{mM} = v_{Sm} + v_o = v_p \sin(\omega t + \phi_{vp} - \frac{2\pi}{3})
\]

\[
+ v_n \sin(-\omega t + \phi_{vn} - \frac{2\pi}{3})
\]

\[
+ v_0 \sin(\omega t + \phi_o).
\]

The subsequent instantaneous powers per phase cluster are evaluated according to [34] as:

\[
P_m = v_{mol}\forall m = a, b, c,\text{ for star configuration}
\]

and the average phase power is obtained by multiplying (3) and (4) and result taking time averaging, as given by:

\[
P_m = \frac{1}{2} \left[ V_p I_p \cos(\phi_{vp} - \phi_{ip}) + V_n I_n \cos(\phi_{vn} - \phi_{in}) \right]
\]

\[
+ \frac{1}{2} V_o I_o \cos(\phi_o - \phi_{vp}) + \sqrt{3} V_p I_p \sin(\phi_{vp} + \phi_{ip})
\]

\[
- \frac{1}{2} V_o I_o \sin(\phi_o - \phi_{vp}) - \sqrt{3} V_n I_n \sin(\phi_{vn} + \phi_{in})
\]

\[
+ \frac{1}{2} V_o I_o \sin(\phi_o - \phi_{vn}) - \sqrt{3} V_o I_o \sin(\phi_o + \phi_{in})
\]

\[
+ \frac{1}{2} V_n I_n \cos(\phi_o + \phi_{vn}) - r \sqrt{3} V_o I_o \sin(\phi_o - \phi_{vp})
\]

\[
+ \frac{1}{2} V_n I_n \cos(\phi_o + \phi_{vn}) - r \sqrt{3} V_o I_o \sin(\phi_o + \phi_{vn})]
\]

\[
P_{cm} = \frac{1}{2} \left[ P_{cm}^{++} + P_{cm}^{--} + P_{cm}^{+} + P_{cm}^{-} \right]
\]

where, \(q = -2, 1, 1\) and \(r = 0, +1, -1\) \(\forall m = a, b, c\).

The first two terms on the RHS in (6) are the products of positive sequence voltage and current (i.e. \(P_{cm}^{++}\)), and negative sequence voltage and current (i.e. \(P_{cm}^{--}\)); these are supplied by the grid to compensate for power losses across the converter. The remaining eight terms can be grouped into two sets; the first four are the cross products of both positive sequence voltage and negative sequence current (i.e. \(P_{cm}^{-}\)), and negative voltage and positive sequence current (i.e. \(P_{cm}^{+}\)). The second set is due to zero sequence voltage; which is the products of zero sequence voltage and positive or negative sequence currents (i.e. \(P_{cm}^{o+}\), and \(P_{cm}^{o-}\)). They can be simplified as:

\[
P_{cm} = \frac{1}{2} \left[ P_{cm}^{++} + P_{cm}^{--} + P_{cm}^{+} + P_{cm}^{-} \right]
\]
Fig. 2. Diagram of cluster voltage balancing control.

The key to preventing cluster dc voltage drifting is to maintain \( P_{Cm} \) to be zero, this requires determining the single unknown zero sequence voltage \( v_o \).

2) Derivation of Zero Sequence Voltage and its third harmonic: A new method for estimating zero sequence voltage is proposed here as shown in Fig. 2 which will be shown giving more accurate result. In this not only does active powers, \( P_{Cm}^+ \) and \( P_{Cm}^- \) obtained via real-time measurement, are used for feed-forward control, combining with three PI regulators for each phase limb dc voltage control, it also applies the unbalanced voltages and converter reference currents. The differences of the feed-forward from the feedback terms give per phase zero sequence average active power \( (P_{Cm}^0) \) as:

\[
P_{Cm}^0 = \frac{P_{Cm}}{P_{Cm,FF}} - \frac{(P_{Cm}^+ - P_{Cm}^-)}{P_{Cm,FF}} \forall m = a, b, c \in \text{star} \tag{8}
\]

The three phase active power is then transformed into \( \alpha-\beta \) form relating to zero sequence voltage, thus:

\[
\begin{bmatrix}
P_{Cm,\alpha} \\
P_{Cm,\beta}
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
1 & -\frac{1}{2} & \frac{1}{2} \\
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}
\end{bmatrix} \begin{bmatrix}
P_{Cm,\alpha} \\
P_{Cm,\beta}
\end{bmatrix}
\]

\[= \frac{1}{2} \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
V_o \cos \varphi_o \\
V_o \sin \varphi_o
\end{bmatrix} \tag{9}
\]

where:
\[
A_{11} = I_p \cos \varphi_{ip} - I_n \cos \varphi_{in}
A_{12} = I_p \sin \varphi_{ip} + I_n \sin \varphi_{in}
A_{21} = -I_p \sin \varphi_{ip} + I_n \sin \varphi_{in}
A_{22} = I_p \cos \varphi_{ip} + I_n \cos \varphi_{in}
\]

The zero sequence voltage can then be expressed as:

\[
\begin{bmatrix}
V_o \cos \varphi_o \\
V_o \sin \varphi_o
\end{bmatrix} = \frac{2}{I_p^2 - I_n^2} \begin{bmatrix}
A_{22} & -A_{12} \\
-A_{21} & A_{11}
\end{bmatrix} \begin{bmatrix}
P_{Cm,\alpha} \\
P_{Cm,\beta}
\end{bmatrix} \tag{11}
\]

where the magnitude and phase angle of the zero sequence voltage and current are given as:

\[
V_o = \sqrt{(V_o \cos \varphi_o)^2 + (V_o \sin \varphi_o)^2}
\]

\[
\varphi_o = \arctan \left( \frac{V_o \sin \varphi_o}{V_o \cos \varphi_o} \right) \tag{12}
\]

Subsequently its third harmonic element is:

\[
V_3 = \frac{1}{6} \sqrt{(V_o \cos \varphi_o)^2 + (V_o \sin \varphi_o)^2} \sin (3\omega t + 3\varphi_o) \tag{13}
\]

It can be seen from (11) that for the SSBC-MMCC, the zero sequence voltage becomes infinite \((i.e. I_p = I_n)\), consequently resulting in one or two converter phase voltages requiring unlimitedly high cluster dc-link voltage which is not realisable. Thus the capability of a SSBC-MMCC STATCOM for unbalanced load compensation is limited by the rating of its phase cluster voltages.

B. SDBC-MMCC with Third Harmonic Current Injection

For SDBC-MMCC the injected current term is given as:

\[
i_{m3} = I_o (\sin(\omega t + \varphi_o) + \frac{1}{6} \sin(3\omega t + 3\varphi_o)). \tag{14}
\]

1) Power Imbalance Analysis: To evaluate just injecting zero sequence circulating current, the phase cluster voltages, the converter reference currents are given respectively as follows:

\[
v_m = \sqrt{3} [V_p \sin(\omega t + \varphi_{vp} - k \frac{2\pi}{3} + \frac{\pi}{6}) + V_n \sin(-\omega t + \varphi_{vn} - k \frac{2\pi}{3} + \frac{\pi}{6})]. \tag{15}
\]

and

\[
i_{m0} = i_m + i_o = \frac{1}{\sqrt{3}} [I_p \sin(\omega t + \varphi_{ip} - k \frac{2\pi}{3} + \frac{\pi}{6}) + I_n \sin(-\omega t + \varphi_{in} - k \frac{2\pi}{3} + \frac{\pi}{6})] + I_o \sin(\omega t + \varphi_o). \tag{16}
\]

where \( \varphi_{vp}, \varphi_{in} \) are respectively the phase angles of the positive and negative sequence currents, and \( k = 0, 1, 2 \) and for \( m = ab, bc, ca \) (delta) respectively.

Multiplying \( v_m \) and \( i_{m0} \) gives the equation for instantaneous power per phase cluster which are the same as \( \text{(6)} \) and \( \text{(7)} \) except phase index \( m = ab, bc, ca \). Note that the last two terms in \( \text{(7)} \), \( P_{Cm,FP} \) and \( P_{Cm,FF} \), are now due to zero sequence current multiplying respectively positive or negative sequence voltages. Similar to star connection case, preventing cluster dc voltage drifting is to maintain \( P_{Cm} \) to be zero, by using the single zero sequence current \( i_o \) circulating in the converter clusters.

2) Derivation of Zero Sequence Current and its third harmonic: This follows the same principle as that used in the SSBC case, hence the control block diagram shown in Fig. 2 is applicable here. The per phase zero sequence average active power \( (P_{Cm}^0) \) can also be evaluated by the differences between the feed-forward and feedback powers given by \( \text{(8)} \) and by applying \( \alpha-\beta \) transformation, it can be expressed as:

\[
\begin{bmatrix}
P_{Cm,\alpha} \\
P_{Cm,\beta}
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
1 & -\frac{1}{2} & \frac{1}{2} \\
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}
\end{bmatrix} \begin{bmatrix}
P_{Cm,\alpha} \\
P_{Cm,\beta}
\end{bmatrix} \tag{17}
\]

where:
\[
\begin{bmatrix}
P_{Cm,\alpha} \\
P_{Cm,\beta}
\end{bmatrix} = \frac{\sqrt{3}}{4} \begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix} \begin{bmatrix}
I_o \cos \varphi_o \\
I_o \sin \varphi_o
\end{bmatrix}
\]

\[
B_{11} = \sqrt{3} (V_p \cos \varphi_{vp} - V_n \cos \varphi_{vn}) - (V_p \sin \varphi_{vp} - V_n \sin \varphi_{vn})
B_{12} = (V_p \cos \varphi_{vp} + V_n \cos \varphi_{vn}) + \sqrt{3} (V_p \sin \varphi_{vp} + V_n \sin \varphi_{vn})
B_{21} = (-V_p \cos \varphi_{vp} + V_n \cos \varphi_{vn}) - \sqrt{3} (V_p \sin \varphi_{vp} - V_n \sin \varphi_{vn})
B_{22} = \sqrt{3} (V_p \cos \varphi_{vp} + V_n \cos \varphi_{vn}) - (V_p \sin \varphi_{vp} + V_n \sin \varphi_{vn}) \tag{18}
\]

\[\]
Hence the zero sequence current can then be written as:

\[
\begin{bmatrix}
I_o \cos \phi_o \\
I_o \sin \phi_o
\end{bmatrix} = \frac{2}{\sqrt{3}(V_p^2 - V_n^2)} \begin{bmatrix} B_{22} & -B_{12} \\ -B_{21} & B_{11} \end{bmatrix} \begin{bmatrix} P_o \\
0 \end{bmatrix} \tag{19}
\]

where the magnitude and phase angle of the zero sequence current is given as:

\[
I_o = \sqrt{(I_o \cos \phi_o)^2 + (I_o \sin \phi_o)^2}
\]

\[
\phi_o = \arctan \left( \frac{I_o \sin \phi_o}{I_o \cos \phi_o} \right) \tag{20}
\]

As can be observed in (19), the zero sequence current circulating between the phases increases with the magnitude of \( I_p \) and \( I_n \), but does not tend to infinity when \( I_p = I_n \).

C. Evaluation of Zero Sequence Components

The magnitudes of the zero sequence components vary not only with the \( I_n \) magnitude, but also on its phase relative to \( V_p \). To evaluate this dependence the power losses in the converters will be neglected; and the phase angle of \( I_p \) relative to \( V_p \) is \( +\frac{\pi}{2} \) due to reactive power compensation. The expressions for the zero sequence voltage magnitude and phase angle are:

\[
V_o = \frac{V_p I_p}{I_p^2 - I_n^2} \sqrt{I_p^2 + I_n^2 + 2I_p I_n \cos(\phi_{ip} + 3\phi_{in})} \tag{21}
\]

\[
\phi_o = \arctan \left( \frac{I_o \sin(\phi_{ip} + \phi_{in}) - I_o \sin(2\phi_{in})}{I_o \cos(\phi_{ip} + \phi_{in}) + I_n \cos(2\phi_{in})} \right) \tag{22}
\]

It can be seen from (21) that the magnitude of the zero sequence voltage depends on the magnitude and phase angles of both positive and negative sequence currents to be compensated. Two examples are used to examine the implications of this.

**Case 1:** \( I_p = 1 \text{pu} \), \( I_n = 0.5 \text{pu} \), the positive and negative sequence compensated currents are in phase (i.e. \( \phi_{ip} = \phi_{in} = +\frac{\pi}{2} \)), and \( V_p = 1 \text{pu} \). Then:

\[
V_o = \frac{0.5}{1^2 - 0.5^2} \sqrt{1^2 + 0.5^2 + \cos(2\pi)} = 0.5 \left( \frac{0.75}{2.25} \right) = 1\text{pu}
\]

\[
\phi_o = \arctan \left( \frac{\sin(\pi) - 0.5\sin(\pi)}{\cos(\pi) + 0.5\cos(\pi)} \right) = \arctan \left( \frac{0}{-1.5} \right) = 180^\circ
\]

**Case 2:** \( I_p = 1 \text{pu} \), \( I_n = 0.5 \text{pu} \), the positive and negative sequence compensated currents are anti-phased to each other (i.e. \( \phi_{ip} = \frac{\pi}{2} \), \( \phi_{in} = -\frac{\pi}{2} \)), and \( V_p = 1 \text{pu} \). Then:

\[
V_o = \frac{0.5}{1^2 - 0.5^2} \sqrt{1^2 + 0.5^2 + \cos(-\pi)} = 0.5 \left( \frac{0.75}{2.25} \right) = 0.33\text{pu}
\]

\[
\phi_o = \arctan \left( \frac{\sin(0) - 0.5\sin(0)}{\cos(0) + 0.5\cos(0)} \right) = \arctan \left( \frac{0}{0.5} \right) = 0^\circ
\]

These examples confirm that the phase angles of \( I_p \) and \( I_n \) influence the \( V_o \) magnitude. The 3-D plot in Fig.3 shows the general relationship of \( V_o \) with respect to both \( \phi_{in} \) and \( I_n/I_p \) and further validates the above analysis. The diagram shows that the maximum \( V_o \) value not only occurs at \( \phi_{in} = \frac{\pi}{2} \) but also at \( \phi_{in} = -\frac{\pi}{6} \) and \( -\frac{5\pi}{6} \), and the minimum occurs at \( \phi_{in} = -\frac{\pi}{2} \), \( -\frac{\pi}{6} \) and \( \frac{2\pi}{3} \).
Fig. 5. SSBC converter cluster voltage waveforms (a) injecting zero sequence voltage, (b) zero sequence voltage + its third harmonic component for $I_n/I_p=0.3\, \text{pu}$.

Fig. 6. SDBC converter cluster current waveforms (a) injecting zero sequence current and (b) zero sequence current + its third harmonic component for $I_n/I_p=0.55\, \text{pu}$.

significantly lower than that when using the only sinusoidal zero sequence current.

E. The Control Scheme

Fig. 7 shows the control structure of the STATCOM, which applies to either a star or a delta configured MMCC for unbalanced load compensation. This comprises four sections as shown: the extraction unit, overall dc voltage control, inter-cluster dc voltage balancing control, and current control plus PWM unit. The extraction unit as shown in Fig. 8 is used in extracting the positive and negative sequence components of the grid voltage and load currents for synchronization and also reference current generation for the current control loop. The overall dc capacitor voltage control is used in to generate the necessary positive sequence d-component current $I^+d$ for maintaining the average dc capacitor voltages at their desired values to compensate. The inter-cluster voltage balancing control is required to maintain equality of the three phase cluster voltages. The $\alpha$-$\beta$ active cluster power derived from the output of the cluster balancing control loop shown in Fig. 2 is fed to determine the zero sequence components (i.e. either fundamental or third harmonic components) depending on the MMCC configuration. For inter-cluster voltage balancing control of SDBC MMCC, the zero sequence current components from Fig. 2 are compared with the measured average current $(i_{Cab} + i_{Cbc} + i_{Cac})/3$, and the errors multiplied by a proportional gain $K_{pio}$ are converted into voltage commands. The output signals from this part are combined with the reference voltages generated from the current control loop, hence becoming the reference signals for the PWM unit. For synthesising gate signals for controlling the converter switches the phase-shifted PWM scheme is used [35].

IV. QUANTIFICATION OF OPERATING RANGE EXTENSION

It was shown in Section III(D) that by injecting combined zero sequence voltage or current and its respective third harmonic component for unbalanced load compensation, the peak phase voltage (for star) and current (for delta) are significantly lower than when using zero sequence components only, and hence the range of unbalanced current compensation is extended. However the results in Section III(D) are only for one particular case of load unbalance, the analysis below evaluates the peak zero sequence voltage injected for SSBC case, and current for SDBC for both fundamental and third harmonic injection techniques, while the degree of load current unbalance varies from 0 to 100%, ( i.e. $K_{ir}$ from 0 to 1). The required maximum phase cluster dc-link voltage and cluster current for both star and delta case respectively can be evaluated and compared, and the more effective method should ideally give lower voltage/current ratings.

A. SSBC MMCC

In determining the phase cluster dc-capacitor voltage required by the SSBC-MMCC for load unbalance compensation, the voltages of each phase are dependent on the their cluster individual module dc-link voltages $V_{dc}$, and their total value $\Sigma V_{dc} = n_{mp}.V_{dc}$, where $n_{mp}$ is the number of modules per phase. The three phase voltages may be different and the peak values of the maximum phase voltages can be defined as:

$$V_{dc,\text{rated}} = \text{Max}([V_{m,M}]) \leq n_{mp}V_{dc} \quad (26)$$

where the converter output voltage is given as:

$$V_{m,M} = v_m + v_o + L\frac{dv_m}{dt} + Rv_m = V_{m,M}\sin(\omega t + \phi_m) \quad (27)$$

and $V_f$ defines the voltage drop across the MMCC filter; $m = a, b, c$.

Fig. 9 shows a logarithmic plot of the $V_{dc,\text{rated}}$ varying according to $K_{ir}$, when injecting both zero sequence voltage fundamental plus the third harmonic component. It is seen that by injecting $v_{03}$ for inter cluster voltage balancing, the maximum cluster dc-link voltage, $V_{dc,\text{rated}}$, is clearly lower than when injecting only $v_i$ for $0 \leq K_{ir} \leq 0.45$ and $0.55 \leq K_{ir} \leq 0.9$. Within the range $0.45 \leq K_{ir} \leq 0.55$ a dead zone exists, meaning that the $V_{dc,\text{rated}}$ values of both $v_o$ and $v_{03}$ injection techniques are almost equal. This results from the fact that within $0.45 \leq K_{ir} \leq 0.55$ both $v_o$ and $v_{03}$ are almost equal as illustrated in Fig. 9, but this only occurs for the worst
Fig. 7. MMCC based STATCOM control system.

Fig. 8. Positive and negative sequence extraction.

condition i.e. when $\varphi_F = \varphi_{in}$. Thus the $v_{o3}$ technique requires lower voltage compared to the $v_o$ counterpart. For example, for $K_{ir} = 0.2$, the normalized peak $V_{dc,\text{rated}}/V_p$ under $v_o$ injection is about 1.18pu whereas for $v_{o3}$ it is only 1.07pu. This implies that for an 11kV distribution system compensating $K_{ir} = 0.2$ of load unbalance where each converter module is rated 400V, 33 modules would be required for both $v_o$ injection technique, whereas with $v_{o3}$ injection only 30 modules are required. It is therefore seen that the use of the zero sequence voltage plus its third harmonic reduces the footprint size and cost of the SSBC-STATCOM. The difference in the estimated dc-link voltage between both injection techniques is clearly shown in Fig. 10 except in the region between $0.45 \leq K_{ir} \leq 0.55$ when both techniques are equal.

B. SDBC MMCC

Comparing the current magnitudes when injecting $i_o$ and $i_{o3}$ as a function of $K_{ir}$, the one under $i_{o3}$ is lower than that when injecting its fundamental frequency counterpart as $K_{ir}$ varies from 0 to 1 as seen in Fig. 11.

The phase cluster currents in the SDBC are expressed as:

$$i_m = i_{mp} + i_{mn} + i_o \text{where } m = ab, bc, ca \quad (28)$$

The current rating $I_{\text{rated}}$ of the SDBC is determined by the maximum phase current given as:

$$I_{\text{rated}} = \text{Max}(|I_m|) \quad (29)$$

Fig. 9. Comparison between sinusoidal and third harmonic zero sequence injection with respect to estimated DC-link cluster voltage.

The maximum phase current of the SDBC only occurs when the zero sequence current is in phase with any of the phase cluster currents. Based on (28), this occurs at $\varphi_{in} = -\frac{\pi}{3}$, $-\pi$ and $+\frac{\pi}{3}$.

Fig. 12, shows the relationship between the maximum cluster current and the degree of unbalance $K_{ir}$ for both the $i_o$ and $i_{o3}$ injections at $\varphi_{in} = -\frac{\pi}{3}$. It is seen that by injecting $i_{o3}$
V. EXPERIMENTAL RESULTS AND DISCUSSIONS

The theoretical analysis presented above has been verified by experimental tests for both star and delta configurations. The experimental MMCC prototype built at the University of Leeds laboratory is shown in Fig. 13. Each of the three phase clusters of the MMCC consists of two series connected modules which are three-level full-bridge flying capacitor converters as shown in Fig. 1(d). Hence there are six modules giving altogether 48 IGBT-diode switching pairs.

The parameters and component values of this experimental MMCC converter are given in Table I. The switching scheme used is the multilevel phase-shifted PWM [35] with carrier frequency of 750 Hz. With two cascaded 3-level flying capacitor modules per phase, the effective lowest order harmonic frequency on the phase voltage is \((120 \pm 1) \times 50\) Hz = \((6000 \pm 50)\) Hz. The filter parameters chosen are \(1 mH, 1 \Omega\) which are effective in eliminating switching harmonics. The submodule capacitors were selected to ensure that their voltage ripples do not exceed \(\pm 10\%\) of its nominal voltage rating of 400 V. The principle of the design calculation is given in [36]. The controller gain parameters of the overall dc-link, cluster and circulating current \(K_{p_{dc}} = 0.5, K_{i_{dc}} = 10, K_{p} = 1, K_{i} = 10\) and \(K_{pio} = 30\) are selected to ensure zero steady state errors, less than \(10\%\) overshoots and fast transient responses.

The control scheme is implemented using a digital signal processor (DSP-TMS320C6713) which calculates the converter three-phase voltage reference values. For PWM signal generation a Field Programmable Gate Array (FPGA) (Actel ProAsic III) is used which is connected to the DSP external memory interface. Pulse signals are applied to the switches through fibre-optical cables.

For these tests the load was set to have a degree of load unbalance up to \(K_{ir} = 0.7\). Figs. 14, 15 and 16 show the results obtained for the SSBC-MMCC, while results in Figs. 17, 18 and 19 are for the SDBC-MMCC. For each figure,
the phase voltage and current waveforms are denoted as blue, green and red for phases $a$, $b$, $c$ for star and $ab$, $bc$, and $ca$ for delta connections.

Fig. 14 shows the waveforms for the SSBC-based STATCOM for reactive power compensation under unbalanced load conditions while sinusoidal zero sequence voltage is injected. For comparison, Fig. 15 gives the corresponding waveforms under the same operating conditions but $v_{o3}$ is injected for cluster dc-voltage balance. As can be seen from both figures, during the time interval $0 < t < 0.1s$, the STATCOM operates under balanced load condition for reactive power compensation, so both $v_a$ and $v_{o3}$ are zero as shown in Fig. 14(f) and Fig. 15(f). The measured converter terminal voltages for SSBC converter under different $K_{ir}$ values are shown in Fig. 16. When $K_{ir} = 0$, load current and terminal voltages are balanced as seen from Figs. 14(a), 15(a) and 16(a). From $t = 0.1s$, load current imbalance occurs and the level of imbalance measured by $K_{ir}$ increases from 0.21 to 0.65 in steps of 0.21, 0.105, 0.075 and 0.05 as shown in Fig. 14 for $v_o$ injection. In Fig. 15, for $v_{o3}$ injection, $K_{ir}$ rises from 0.21 to 0.7 in steps of 0.21, 0.14, 0.09 and 0.05. The STATCOM controller compensates load unbalance in both cases.

It can be seen that when only injecting $v_o$, negative sequence current can be compensated for $K_{ir}$ up to 0.6 ($0.1 < t < 0.5sec$) (see Fig. 14(c)) and the dc-link capacitor voltages are maintained within their nominal ratings (see Fig. 14(e)). Fig. 16(b) shows the converter three terminal voltages when $K_{ir} = 0.42$, which are within the converter linear modulation range. However, further increasing the level of current imbalance to $K_{ir} = 0.65 (0.5 < t < 0.6sec)$, the converter phase voltages, especially phase C voltage, rise fast hence exceeding the linear modulation range as seen in Fig. 14(c). This is due to high zero sequence voltage (reaching 60V) required as seen in Fig. 14(f). The dc-link capacitor voltages become uncontrollable because the phase limb voltages required to maintain the converter operation at $K_{ir} = 0.65$ are significantly over their limit (see Fig. 14(c)), thus resulting in distorted currents injecting into the grid as shown in Fig. 14(d). Fig. 16(c) shows the converter terminal voltages at $K_{ir} = 0.65$ which are severely distorted and unbalanced.

In contrast, using the third harmonic zero sequence voltage $v_{o3}$ injection, the operating range is extended as shown in Fig. 15. For $K_{ir} = 0.65$ within the time interval $0.4 < t < 0.5sec$, the converter reference and terminal voltages, particularly phase C voltage, are all within the rated limit as seen in Fig. 15(c) and Fig. 16(d). Also, the dc-link voltage can be maintained balanced (see Fig. 15(e)). Thus it is clear that, using the third harmonic voltage injection technique, the SSBC-MMCC based STATCOM is able to operate normally for higher level of load current imbalance compared to the case when only fundamental zero sequence voltage $v_o$ is used.

For comparison, the performance of the SDBC-MMCC based STATCOM under unbalanced load compensation is also investigated, and the results are as illustrated in Figs. 17 and 18. The superiority of the SDBC over the SSBC is mostly evident when the level of load imbalance is high. For example when $K_{ir} = 0.70$, the SDBC can compensate unbalanced current well, the sub-module capacitor voltages are maintained within 10% of their nominal rated values (see Fig. 17(e) and 18(e)). In this case the results obtained from using zero sequence current $i_o$ and third order harmonic plus fundamental zero sequence $i_{o3}$ injections are also compared, the magnitude of $i_{o3}$ required at $K_{ir} = 0.70$ is 0.1A lower than its counterpart which is 0.6A (see Fig. 17(f) and 18(f)). The converter terminal voltage waveforms are shown in Fig. 19 for $K_{ir}$ equal to 0 and 0.70 respectively. From these waveforms, it is seen that for SDBC-MMCC, increase in the level of load imbalance does not result in large increment of the converter voltages.
VI. CONCLUSION

This paper has proposed a new harmonic injection technique for star or delta configured MMCC-STATCOMs to balance their inter-cluster dc-link voltages when they are used for unbalanced load compensation. The theoretical and experimental contributions of this work can be summarized as follows:

1) Injecting a harmonic voltage for a star connected STATCOM reduces the maximum value of dc-link voltage, and injecting a harmonic current for a delta connected STATCOM reduces the maximum values of converter phase circulating current, compared to applying only sinusoidal zero sequence components, at a given degree of load unbalance.

2) This reduction improves the STATCOM capability for compensating unbalanced load current. Experimental results show that the SSBC-STATCOM is enabled to compensate unbalanced load up to 0.65, compared with the sinusoidal zero sequence voltage which can only cope with 0.60, while the converter phase cluster voltage for the latter is 12% higher.

3) Both SSBC and SDBC STATCOMs can compensate simultaneously the positive sequence reactive power and negative sequence active and reactive power. The SDBC-STATCOM is superior in unbalanced load compensation compared to its SSBC counterpart, and it can compensate up to 1.0 of load unbalance, $K_{ir}$.

REFERENCES

Recent advances and industrial applications of multi-level converters., IEEE Transactions on industry applications, 1996 May; 32(3), pp. 509-517.


Sano, Kenichiro, and Masahiro Takasaki, A transformerless D-STATCOM based on a multivoltage cascade converter requiring no DC sources, IEEE transactions on power electronics 27.6 (2012): 2783-2795.


Gultekin, Burhan, and Muammer Ermis., Cascaded multilevel converter-based transmission STATCOM: System design methodology and development of a 12 kV ±12 MVAr power stage., IEEE transactions on power electronics 28.11 (2013): 4930-4950.


