Peak-to-Average Power Ratio Reduction of OFDM Signals Using PTS Scheme With Low Computational Complexity

Jun Hou, Jianhua Ge, and Jing Li

***Abstract*—Partial transmit sequences (PTS) is one of the most attractive schemes to reduce the peak-to-average power ratio (PAPR) in orthogonal frequency division multiplexing (OFDM) systems. However, the conventional PTS scheme requires an exhaustive searching over all combinations of allowed phase factors. Consequently, the computational complexity increases exponentially with the number of the subblocks. By utilizing the correlation among the candidate signals generated in PTS, a novel scheme is proposed to decrease the computational complexity. The performance analysis shows that the proposed scheme can reduce the computational complexity dramatically while achieving the same PAPR reduction compared to the conventional PTS scheme.**

***Index Terms*—Computational complexity, orthogonal frequency division multiplexing (OFDM), partial transmit sequences (PTS), peak-to-average power ratio (PAPR).**

I. INTRODUCTION

AS A MULTICARRIER modulation technique, orthogonal frequency division multiplexing (OFDM) [1], [2] has gained popularity in a number of applications including digital audio broadcasting (DAB), terrestrial digital video broadcasting (DVB-T), the IEEE 802.11a standard for wireless local area networks (WLAN) and the IEEE 802.16d standard for wireless metropolitan area networks (WMAN), owing to its high bandwidth efficiency and robustness to multipath fading.

However, some drawbacks are still unresolved in the design of OFDM system. One of the major problems of OFDM system is the high peak-to-average power ratio (PAPR), which may result in significant distortion when the transmitted signals passed through a nonlinear device such as the power amplifier. To deal with this problem, many PAPR reduction schemes have been proposed, such as block coding [3], clipping [4], companding transform schemes [5]–[9], selective mapping (SLM) [10]–[12], and partial transmit sequence (PTS) [13]–[21]. Among which, PTS is a distortionless phase optimization scheme that provides excellent PAPR reduction with a small amount of redundancy. In PTS, an input data sequence is divided into a number of disjoint subblocks [13], which are then weighted by a set of phase factors to create a set of candidate signals. Finally, the candidate with the lowest PAPR is chosen for transmission.

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Nevertheless, finding the optimum candidate requires the exhaustive search over all combinations of allowed phase factors, and the search complexity increases exponentially with the number of subblocks. To reduce the computational complexity, some modified techniques have been presented [14]–[19]. Most of them focus on reducing the number of candidate signals. For example, iterative flipping with a preset threshold algorithm [18] has been introduced to reduce the PAPR with less complexity and easier implementation. However, the combination of its phase factors is suboptimal and there is some performance gap between the conventional PTS scheme and the iterative flipping algorithm.

In this paper, considering the correlation among the phase factors, we present a novel PTS scheme to reduce the computational complexity. In addition, the proposed scheme mainly focuses on simplifying the computation for each candidate signal, instead of reducing the number of candidate signals. Since the number of candidates is not reduced, it can achieve the same PAPR reduction compared with the conventional PTS scheme.

The rest of this paper is organized as follows. Section II formulates the problem of high PAPR and describes the principle of the PTS-based OFDM system. Section III puts forth the proposed low complexity PTS scheme. The computational complexity and PAPR performance of the proposed scheme are evaluated in Section IV and it is followed by a conclusion in Section V.

II. OFDM SYSTEM WITH PTS TO REDUCE PAPR

Let N denote the number of subcarriers used for parallel information transmission and $X_k (0 \leq k \leq N-1)$ represent the kth complex modulated symbol in a block of N information symbols.

The outputs x_n of the N-point inverse discrete Fourier transform (IDFT) of X_k are given by

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \exp\left(\frac{j \cdot 2\pi kn}{N}\right). \tag{1}$$

where $j^2 = -1$. Accordingly, the PAPR of the OFDM signal, defined as the ratio of the maximum power divided by the average power of the signal, is expressed as

$$PAPR = 10 \cdot \log_{10} \frac{Max\left\{|x_n|^2\right\}}{E\left[|x_n|^2\right]} (dB), \tag{2}$$

where $|x_n|$ returns the magnitude of x_n and $E[\cdot]$ denotes the expectation operation. The peak power occurs when the N modulated symbols are added with the same phase.

As shown in Fig. 1, for PAPR reduction using PTS scheme, the frequency domain vector X is partitioned into disjoint V

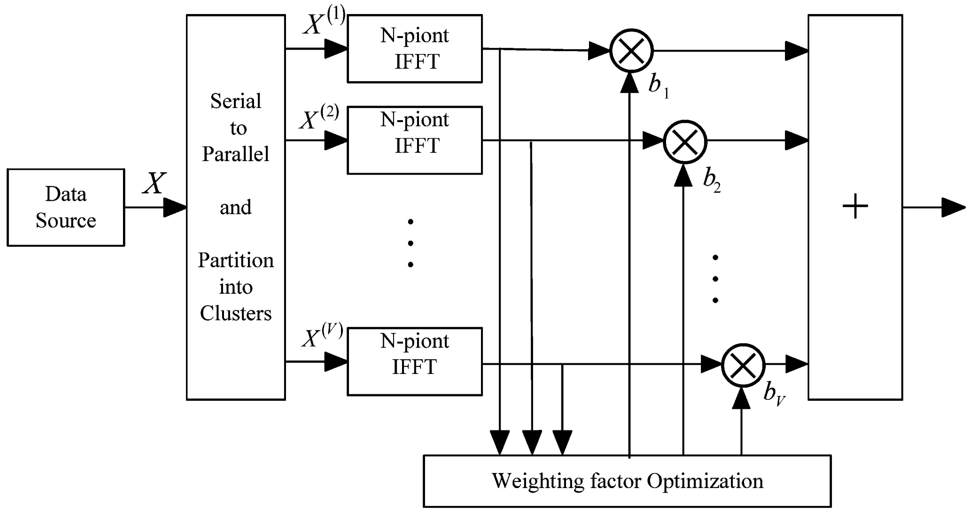

Fig. 1. Block diagram of partial transmit sequence scheme.

subblocks, which are represented by $\{X^{(v)}, \mathrm{v} = 1, 2, \cdots, V\}$. Hence,

$$X = \sum_{v=1}^{V} X^{(v)}, \tag{3}$$

where $X^{(v)} = [X_0^{(v)} X_1^{(v)} \cdots X_{N-1}^{(v)}]$ with $X_k^{(v)} = X_k$ or 0 $(1 \le v \le V)$. Let $\mathbf{b} = \{b_v = e^{j\theta v}, \mathrm{v} = 1, 2, \cdots, V\}$ be the set of phase factors which are applied to the subblocks $X^{(v)}$. The substitute frequency domain signals are

$$X' = \sum_{v=1}^{V} b_v X^{(v)}, (b_v = e^{j\theta_v}, v = 1, 2, \cdots, V). \tag{4}$$

Note that these partial sequences are independently rotated by phase factors $\mathbf{b}$. Taking the IDFT of (4) and using the linearity property of the IDFT, the time domain partial transmit sequences can be expressed as

$$x' = IDFT(X') = \sum_{v=1}^{V} b_v IDFT(X^{(v)}) = \sum_{v=1}^{V} b_v x^{(v)}. \tag{5}$$

The objective is to optimally combine the V subblocks to obtain the time domain OFDM signals with the lowest PAPR. Without any loss of performance, one can set $b_1 = 1$ and observe that there are $(V - 1)$ subblocks to be optimized. Consequently, to achieve the optimal phase factor for each input data sequence (assume that there are W phase factors in the phase set), W^{V-1} combinations should be checked in order to obtain the minimum PAPR. Therefore, the search complexity for an optimum set of the phase factors increases exponentially with the number of subblocks.

III. Proposed PTS Scheme With Low Complexity

In this section, a novel PTS scheme is presented based on listing the phase factors into multiple subsets table and utilizing the correlation among phase factors in each subset, in order to reduce the computational complexity.

Here, we firstly introduce the concept 'basis vector of the phase factors': For a PTS-based OFDM system with $V = 2$ and $W = 2$, we can list all phase factors: $b_1 = \{1, 1\}, b_2 = \{1, -1\}$. For this reason, we can take $B_1 = \{1, 1\}$ as the basis vector of the phase factors, provided the sign of the elements is not considered. As for $V = 2$ and $W = 4$, all phase factors are $b_1 = \{1, 1\}$, $b_2 = \{1, -1\}$ $b_3 = \{1, j\}$, and $b_4 = \{1, -j\}$, so the basis vectors of the phase factors can be written as $B_1 = \{1, 1\}$, $B_2 = \{1, j\}$ (see j as a real number).

Then, list all phase weighting vectors in the Table subject to the following Rules (we would like to first give the Rules and then take an example to explain them in detail):

1 Find the basis vectors of all phase weighting vectors and put them in the first row, note that only one element in the adjacent basis vectors is different;
2 In each column, the phase weighting vectors have the same basis vector;
3 For the adjacent phase weighting vectors in the same column, only the sign of one element is different;
4 The sign of the last phase weighting vectors in one column is the same as the first phase weighting vectors in the next column.

□

Let $sgn(\cdot)$ be the sign function, $B_{i,j}$ represent the jth element of the basis vector B_i, and $b_{Bi,k,m}$ denote the kth phase weighting vector based on the vector B_i, where m represents this phase weighting vector applied to the mth subblock of the transmitted signal. Now we search the optimal phase weighting vector based on the four Rules. In the paper, we select the phase weighting vector $\mathbf{b} = \{-1, 1\}$ or $\mathbf{b} = \{-1, 1, -j, j\}$, that is, $W = 2$ or 4.

For $V > 2$, $x^{(0)} = x$, list the corresponding Table according to the Rules and let $B_1 = \{1, 1, \cdots, 1\}$. Thereby, $x_{B1,1}$ is given by

$$x_{B1,1} = x^1 + x^2 + \cdots + x^V, \tag{6}$$

In order to reduce the computational complexity, we can make use of the Rule 3 to calculate $x_{B1,2}$ from $x_{B1,1}$,

$$\begin{aligned} x_{B1,2} &= b_{B1,2,1}x^1 + b_{B1,2,2}x^2 + \cdots + b_{B1,2,V}x^V \\ &= sgn(b_{B1,2,1})x^1 + sgn(b_{B1,2,2})x^2 + \cdots + sgn(b_{B1,2,V})x^V \\ &= x_{B1,1} - sgn(b_{B1,1,m}) \cdot 2x^m, b_{B1,2,m} \neq b_{B1,1,m}. \end{aligned} \quad (7)$$

where $x^1, x^2, x^m, \cdots, x^V \in x^{(0)}$. Similarly, $x_{B1,i+1}$ can be expressed as

$$\begin{aligned} x_{B1,i+1} &= b_{B1,i+1,1}x^1 + b_{B1,i+1,2}x^2 + \cdots + b_{B1,i+1,V}x^V \\ &= sgn(b_{B1,i+1,1})x^1 + sgn(b_{B1,i+1,2})x^2 \\ &\quad + \cdots + sgn(b_{B1,i+1,V})x^V \\ &= x_{B1,i} - sgn(b_{B1,i,m}) \cdot 2x^m, b_{B1,i+1,m} \\ &\neq b_{B1,i,m}. \end{aligned} \quad (8)$$

In the same way, $x_{B1,last}(last = 2^{V-1})$ can be calculated.

To calculate $x_{B2,1}$ from $x_{B1,last}$, we introduce a new parameter, $P_{i,j}$, which is defined as: for $W = 2$, $P_{i,j} = 1$; for $W = 4$,

$$P_{i,j} = \begin{cases} 1 & B_{i,j} = B_{i-1,j} \\ -j & B_{i,j} \neq B_{i-1,j}, B_{i,j} = 1 \\ j & B_{i,j} \neq B_{i-1,j}, B_{i,j} = j \end{cases}. \quad (9)$$

Thus, considering the Rule 4, provided that $B_{2,m} \neq B_{1,m}$, we have

$$\begin{aligned} x_{B2,1} &= b_{B2,1,1}x^1 + b_{B2,1,2}x^2 + \cdots + b_{B2,1,V}x^V \\ &\overset{Rule4}{=} x_{B1,last} - b_{B1,last,m}x^m + b_{B1,last,m} \cdot P_{2,m}x^m, \end{aligned} \quad (10)$$

where $x^m \in x^{(0)}$, and $last = 2^{V-1}$. Simultaneously, update x^m and $x^{(0)}$: $x^m_{(\text{new})} = P_{2,m} \cdot x^m$, $x^{(1)} = \{x^1, x^2, \cdots, x^m_{(\text{new})}, \cdots, x^V\}$.

By the same token, according to the Rules 3 and 4, we obtain the following formulas

$$\begin{aligned} & x_{B(k+1),1} \\ &= b_{B(k+1),1,1}x^1 + b_{B(k+1),1,2}x^2 + \cdots + b_{B(k+1),1,V}x^V \\ &= x_{Bk,last} - b_{Bk,last,m}x^m + b_{Bk,last,m} \cdot P_{(k+1),m}x^m, \end{aligned} \quad (11)$$

where $last = 2^{V-1}$, $B_{(k+1),m} \neq B_{k,m}$. Based on $B_{(k+1),m} \neq B_{k,m}$, update x^m and $x^{(k)}$: $x^m_{(\text{new})} = P_{k+1,m} \cdot x^m$, $x^{(k)} = \{x^1, x^2, x^m_{(\text{new})}, \cdots, x^V\}$.

$$\begin{aligned} x_{B(k+1),i+1} &= b_{B(k+1),i+1,1}x^1 + b_{B(k+1),i+1,2}x^2 \\ &\quad + \cdots + b_{B(k+1),i+1,V}x^V \\ &= x_{B(k+1),i} - b_{B(k+1),i,m} \cdot 2x^m, \end{aligned} \quad (12)$$

where $x^m \in x^{(k)}$, $b_{B(k+1),i+1,m} \neq b_{B(k+1),i,m}$.

Repeat this process until all possible phase weighting vectors have been searched. □

It is straightforward to summarize the general form:

For $V > 2$, $W = 2$ or $W = 4$, $x^i \in x, (i = 1, 2, \cdots, V)$: First, list the Rule Table and let $x^{(0)} = x$, $B_1 = \{1, 1, \ldots, 1\}$. Note that, the range of u in $x^{(u)}$ is $0 \leq u \leq (W/2)^{V-1} - 1$. Then, with the aid of Rule 3, we have

$$x_{Bk,i+1} = x_{Bk,i} - b_{Bk,i,m} \cdot 2x^m, \quad (13)$$

where $b_{Bk,i+1,m} \neq b_{Bk,i,m}$, $x^m \in x^{(k-1)}$. Here, $b_{Bk,i,m}$ is equal to 1 or -1.

By combining the Rule 4 and (9), we can obtain $x_{B(k+1),1}$ from $x_{Bk,last}$ as follows

$$x_{B(k+1),1} = x_{Bk,last} - b_{Bk,last,m}x^m + b_{Bk,last,m} \cdot P_{(k+1),m}x^m, \quad (14)$$

where $last = 2^{V-1}$, $x^m \in x^{(k-1)}$, $B_{(k+1),m} \neq B_{k,m}$. Based on $B_{(k+1),m} \neq B_{k,m}$, update x^m and $x^{(k)}$: $x^m_{(\text{new})} = P_{(k+1),m} \cdot x^m$, $x^{(k)} = \{x^1, x^2, x^m_{(\text{new})}, \cdots, x^V\}$. Similar to (13),

$$x_{B(k+1),i+1} = x_{B(k+1),i} - b_{B(k+1),i,m} \cdot 2x^m, \quad (15)$$

where $b_{B(k+1),i+1,m} \neq b_{B(k+1),i,m}$, $x^m \in x^{(k)}$. This algorithm iterates until $x_{B(W/2)^{V-1},2^{V-1}}$ is obtained.

In the same manner, the number of phase weighting vectors is searched and the corresponding PAPR values are computed until all possible phase weighing vectors have been searched. Finally, choose the candidate signal with the minimum PAPR as the optimized transmitted signal. □

Let us summarize the previous deduction. It can be concluded that the optimization search is done by using the given basis vectors and the updated signal $x^{(k)}$. During the calculation from $x_{Bk,i}$ to $x_{Bk,i+1}$, only the complex addition is needed (considering the sign of the corresponding phase weighting vectors). Consequently, the Rule Table can be updated (namely Sign Rule Table) by replacing $b_{Bi,k,m} = sgn(b_{Bi,k,m})$ (see j as a real number), shown in the following example:

Take $V = 3$ and $W = 4$ to search the optimum candidate signal. As seen in Tables I and II,

1 : The basis vectors are B_1, B_2, B_3, and B_4. Compared the vector B_1 with B_2 in the adjacent rows, only the last element is different (1 and j);

2 : The basis vectors of phase weighting vectors in the first column are all $\{1, 1, 1\}$, other columns also have the similar case;

3 : For the adjacent phase weighting vector in the same column, only the sign of one element is different (e.g., $b_{B1,1}$ and $b_{B1,2}$, only the sign of the second element is different);

4 : The sign of elements of $b_{Bk,4}$ is identical with that of $b_{B(k+1),1}$ (see j as a real number).

Let that $x = \{x^1, x^2, x^3\}$, $b_{B1,1} = \{1, 1, 1\}$. Hence, $x_{B1,1}$ is given by

$$\begin{aligned} x_{B1,1} &= b_{B1,1,1}x^1 + b_{B1,1,2}x^2 + b_{B1,1,3}x^3 \\ &= sgn(b_{B1,1,1})x^1 + sgn(b_{B1,1,2})x^2 + sgn(b_{B1,1,3})x^3. \end{aligned} \quad (16)$$

According to the Rule 3, $x_{B1,2}$ can be expressed as

$$\begin{aligned} x_{B1,2} &= b_{B1,2,1}x^1 + b_{B1,2,2}x^2 + b_{B1,2,3}x^3 \\ &= sgn(b_{B1,2,1})x^1 + sgn(b_{B1,2,2})x^2 + sgn(b_{B1,2,3})x^3 \\ &= x_{B1,1} - sgn(b_{B1,1,2}) \cdot 2x^2, b_{B1,2,2} \neq b_{B1,1,2}. \end{aligned} \quad (17)$$

Similarly, $x_{B1,i+1}$ is calculated as

$$\begin{aligned} x_{B1,i+1} &= b_{B1,i+1,1}x^1 + b_{B1,i+1,2}x^2 + b_{B1,i+1,3}x^3 \\ &= sgn(b_{B1,i+1,1})x^1 \\ &\quad + sgn(b_{B1,i+1,2})x^2 + sgn(b_{B1,i+1,3})x^3 \\ &= x_{B1,i} - sgn(b_{B1,i,m}) \cdot 2x^m, b_{B1,i+1,m} \neq b_{B1,i,m}. \end{aligned} \quad (18)$$

TABLE I
PHASE WEIGHTING VECTORS

	phase weighting vectors			
basis vector	B_1={ 1 1 1}	B_2={1 1 j}	B_3={1 j j}	B_4={1 j 1}
phase	$b_{B1,1}$={+1 +1 +1}	$b_{B2,1}$={+1 +1 –j}	$b_{B3,1}$={+1 +j +j}	$b_{B4,1}$={+1 +j –1}
weighting	$b_{B1,2}$={+1 –1 +1}	$b_{B2,2}$={+1 –1 –j}	$b_{B3,2}$={+1 –j +j}	$b_{B4,2}$={+1 –j –1}
vectors	$b_{B1,3}$={+1 –1 –1}	$b_{B2,3}$={+1 –1 +j}	$b_{B3,3}$={+1 –j –j}	$b_{B4,3}$={+1 –j +1}
	$b_{B1,4}$={+1 +1 –1}	$b_{B2,4}$={+1 +1 +j}	$b_{B3,4}$={+1 +j –j}	$b_{B4,4}$={+1 +j +1}

↓ update

TABLE II
SIGN OF PHASE WEIGHTING VECTORS

	sign of phase weighting vectors			
basis vector	B_1={ 1 1 1}	B_2={1 1 j}	B_3={1 j j}	B_4={1 j 1}
phase	$b_{B1,1}$={+ + +}	$b_{B2,1}$={+ + –}	$b_{B3,1}$={+ + +}	$b_{B4,1}$={+ + –}
weighting	$b_{B1,2}$={+ – +}	$b_{B2,2}$={+ – –}	$b_{B3,2}$={+ – +}	$b_{B4,2}$={+ – –}
vectors	$b_{B1,3}$={+ – –}	$b_{B2,3}$={+ – +}	$b_{B3,3}$={+ – –}	$b_{B4,3}$={+ – +}
	$b_{B1,4}$={+ + –}	$b_{B2,4}$={+ + +}	$b_{B3,4}$={+ + –}	$b_{B4,4}$={+ + +}

where $x^m \in x$. In the same way, $x_{B1,4}$ can be calculated.

From the Rule 4 and (9), we have

$$\begin{aligned} x_{B2,1} &= b_{B2,1,1}x^1 + b_{B2,1,2}x^2 + b_{B2,1,3}x^3 \\ &= sgn(b_{B2,1,1})x^1 + sgn(b_{B2,1,2})x^2 \\ &\quad + sgn(b_{B2,1,3}) \cdot P_{2,3}x^3 \\ &\overset{Rule4}{=} sgn(b_{B1,4,1})x^1 + sgn(b_{B1,4,2})x^2 \\ &\quad + sgn(b_{B1,4,3}) \cdot P_{2,3}x^3 \\ &= x_{B1,4} - sgn(b_{B1,4,3})x^3 \\ &\quad + sgn(b_{B1,4,3}) \cdot P_{2,3}x^3, B_{2,3} \neq B_{1,3}. \end{aligned} \tag{19}$$

Then update x^3 and $x^{(1)}$: $x^3_{(\text{new})} = P_{2,3}x^3$, $x^{(1)} = \{x^1, x^2, x^3_{(\text{new})}\}$, note that $B_{2,3} \neq B_{1,3}$. Considering the Rule 3, $x_{B2,i+1}$ can be expressed as

$$\begin{aligned} x_{B2,i+1} &= b_{B2,i+1,1}x^1 + b_{B2,i+1,2}x^2 + b_{B2,i+1,3}x^3 \\ &= sgn(b_{B2,i+1,1})x^1 \\ &\quad + sgn(b_{B2,i+1,2})x^2 + sgn(b_{B2,i+1,3})x^3 \\ &= x_{B2,i} - sgn(b_{B2,i,m}) \cdot 2x^m, \end{aligned} \tag{20}$$

where $b_{B2,i+1,m} \neq b_{B2,i,m}$ and $x^m \in x^{(1)}$.

Similar to (19), $x_{B3,1}$ can be obtained from $x_{B2,4}$,

$$\begin{aligned} x_{B3,1} &= b_{B3,1,1}x^1 + b_{B3,1,2}x^2 + b_{B3,1,3}x^3 \\ &= sgn(b_{B3,1,1})x^1 + sgn(b_{B3,1,2}) \\ &\quad \cdot P_{3,2}x^2 + sgn(b_{B3,1,3})x^3 \\ &\overset{Rule4}{=} sgn(b_{B2,4,1})x^1 + sgn(b_{B2,4,2}) \\ &\quad \cdot P_{3,2}x^2 + sgn(b_{B2,4,3})x^3 \\ &= x_{B2,4} - sgn(b_{B2,4,2})x^2 + sgn(b_{B2,4,2}) \\ &\quad \cdot P_{3,2}x^2, B_{3,2} \neq B_{2,2}. \end{aligned} \tag{21}$$

Update x^2 and $x^{(2)}$: $x^2_{(\text{new})} = P_{3,2}x^2$, $x^{(2)} = \{x^1, x^2_{(\text{new})}, x^3\}$, where $B_{3,2} \neq B_{2,2}$.

In the same way, until $x_{B4,4}$ has been searched. Then, choose the candidate signal with the minimum PAPR as the optimized transmitted signal. □

Set a specific V and W, the Sign Rule Table can be determined in advance without increasing the computational complexity in the transmitter and receiver. By using the general form, we can reduce the computational complexity dramatically and achieve the same PAPR reduction performance compared to the conventional PTS scheme.

IV. PERFORMANCE ANALYSIS

A. Analysis of the Computational Complexity

The main contribution of the proposed scheme is to reduce the computational complexity of the obtaining time domain vector. In this section, we will analyze the computational complexity and PAPR of the proposed scheme. Furthermore, it should be noted that the multiplicative complexity dominates the total computational complexity.

According to (14), the complex multiplication is only used in the case of computing $x_{B(k+1),1}$ from $x_{Bk,last}$. Since the dimension of phase weighting vectors of Sign Rule Table established in Section III is $2^{V-1} \times (W/2)^{V-1}$, it is clear that the number of the complex multiplication operations of the proposed scheme needed in the process of phase weighting combination is given by

$$n_{mul} = N \cdot \left[(W/2)^{V-1} - 1\right], \tag{22}$$

TABLE III
COMPUTATIONAL COMPLEXITY REDUCTION RATIO

Comparison	CCRR (%)	
Phase factor	W=4	
Subblock	V=4	V=6
Complex Add.	67	80
Complex Mul.	92	98

while that of the conventional PTS-based OFDM scheme can be expressed as

$$n_{mul} = N \cdot \left[1 \cdot C_{V-1}^{1}(W/2-1)^{1} + \cdots + k \cdot C_{V-1}^{k}(W/2-1)^{k} + \cdots + (V-1) \cdot C_{V-1}^{V-1}(W/2-1)^{V-1}\right] \cdot 2^{V-1}. \quad (23)$$

Here, note that the complex multiplication is only used when the element of the basis vector of phase weighting vector is unequal to 1. On the other hand, in the conventional PTS scheme, $x_{Bk,i+1}$ is calculated as

$$x_{Bk,i+1} = b_{Bk,i+1,1}x^{1} + b_{Bk,i+1,2}x^{2} + \cdots + b_{Bk,i+1,V}x^{V}. \quad (24)$$

However, in order to reduce the addition computational complexity, the proposed scheme make use of the Rule 3 to calculate $x_{Bk,i+1}$ from $x_{Bk,i}$

$$\begin{aligned} x_{Bk,i+1} &= b_{Bk,i+1,1}x^{1} + b_{Bk,i+1,2}x^{2} + \cdots + b_{Bk,i+1,V}x^{V} \\ &= x_{Bk,i} - b_{Bk,i,m} \cdot 2x^{m}. \end{aligned} \quad (25)$$

For this reason, it can be concluded that computing $x_{Bk,i+1}$ from $x_{Bk,i}$ by (25), the computational complexity of the proposed scheme has been reduced to N additions because it is only necessary to calculate the subblock x^{m} given that we know $x_{Bk,i}$. Thus, the complex addition complexity of the proposed scheme is about the one $(V-1)$th of the conventional PTS scheme.

In [19], the computational complexity reduction ratio (CCRR) of the proposed PTS OFDM scheme over the conventional PTS OFDM scheme is defined as

$$CCRR = \left(1 - \frac{\text{complexity of proposed PTS}}{\text{complexity of conventional PTS}}\right) \times 100\%. \quad (26)$$

Table III gives CCRR of the proposed scheme over the conventional PTS OFDM scheme with typical values of V and W.

As shown in Table III, compared with the conventional PTS, the proposed scheme can significantly reduce the computational complexity. Specially, when the number of subblocks $V = 4$ and $V = 6$, the complex multiplication of the proposed scheme are only about 8% and 2% of the conventional PTS. It also should be noted that the required side information is the same as the conventional PTS scheme.

B. Analysis of the PAPR

To reduce the computational complexity of a PTS-based OFDM system, most authors focus on reducing the number of candidate signals. Therefore, the computational complexity is reduced clearly at the cost of performance loss for PAPR reduction. Unlike these methods, the proposed scheme reduces complexity by using the correlation among the adjacent candidates. Since the number of candidate signals is not reduced, it can achieve the same PAPR reduction as the conventional PTS scheme. As shown in Fig. 2, the results of computer simulation are coincident with the theoretical analysis. Note that the subblock partition method is interleaved partition.

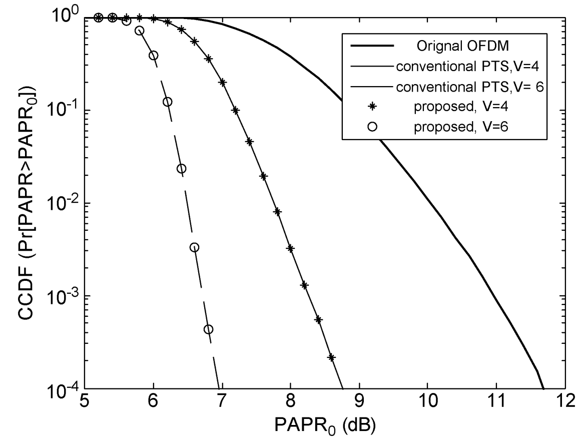

Fig. 2. CCDF's of original OFDM, conventional PTS, and proposed PTS with 256 subcarriers, QPSK modulation, the phase factor $W = 4$, and oversampling factor $L = 4$.

V. CONCLUSION

High PAPR of transmitted signal is one of the major drawbacks of OFDM systems. In the conventional PTS scheme, the computational complexity increases extensively with the number of subblocks. In order to reduce this complexity, a novel PTS scheme has been proposed by utilizing the correlation among the candidate signals. Performance analysis has been shown that the proposed scheme can obtain the same PAPR reduction compared to the conventional PTS while significantly reduce the complexity.

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