2016 International Conference on Computing, Networking and Communications (ICNC), Workshop on Computing, Networking and Communications (CNC)

High Performance Phase Rotated Spreading Codes for MC-CDMA

Zhiping Zhang[1], Yang Qu[1], Lin Zhang[2], John Ellinger[3] and Zhiqiang Wu[2]
[1]Department of Electrical Engineering, Wright State University
[2]School of Information Science and Technology, Sun Yat-sen University
Sensors Directorate, Air Force Research Laboratory[3]
Email: Zhang.176@wright.edu

***Abstract*—In conventional multi-carrier CDMA (MC-CDMA) systems, binary spreading codes such as Hadamard-Walsh codes are employed to spread user information across all subcarriers to exploit frequency diversity in frequency selective fading channels. We first recap that because of the binary nature of the spreading codes, transmission power is not distributed evenly across all subcarriers. Oftentimes, certain subcarriers have zero transmission power, leading to less diversity to be exploited at receiver. We employ a phase rotated spreading code design and derive the corresponding combining scheme for the phase rotated codes. As a direct result, MC-CDMA system now exploits full diversity available in the channel at all times, leading to significant performance gain. Simulation results over various multi-path fading channels confirm the performance gain of the scheme.**

I. Introduction

In multi-carrier code division multiple access (MC-CDMA) systems, unique spreading codes are assigned to different users to spread their information bits over all subcarriers to exploit frequency diversity in frequency selective fading channels [1]- [7]. Conventionally, binary spreading codes such as Hadamard-Walsh codes are employed. With its excellent performance in fading channels and ease of implementation through FFT/IFFT, MC-CDMA has attracted lots of attention in recent years [1]- [7].

It has been recognized in [8] and [9] that due to the nature of the spreading codes being used in conventional MC-CDMA systems, the BER performance of the commonly used Hadamard transform is asymptotically bad. A phase rotated spreading transform was proposed in [8] to achieve better asymptotic performance. However, in previous works the spreading code length is assumed to be quite small (e.g., the length is 8 in [8] and 4 or 8 in [9]). Therefore, either a optimum maximum likelihood detection receiver or a sub-optimum linear equalization detection receiver can be exploited. In most of MC-CDMA systems, it is highly desired to develop a simple yet effective subcarrier combining scheme that offers excellent performance at minimal complexity.

In this paper, we revisit the phase rotated spreading code design for MC-CDMA system and derive the minimized mean square error combining (MMSEC) scheme for it. We show that the MMSEC receiver can effectively exploit full diversity and offer excellent BER performance at minimal computational complexity. The derivation of the MMSEC also indicates that the phase rotated spreading code design reduces the power of multiple access interference (MAI) in half. Additionally, we show that previously developed carrier interferometry (CI) MC-CDMA (CI/MC-CDMA) systems provides phase rotation (and consequently reduced MAI) in the majority of the subcarriers. This offers a new understanding of the performance gain of CI/MC-CDMA systems.

The remaining part of this paper is organized as follows. In Section II, we recap the problem of uneven power distribution in MC-CDMA systems using Hadamard-Walsh codes and review the phase rotated spreading code design. In Section III, we derive the low complexity minimized mean square error combining scheme for the phase rotated spreading code. We also discuss the BER performance gain of CI/MC-CDMA systems and show that this gain is also coming from the phase rotation. In Section IV, we provide numerical results over different conditions to show the benefit. Conclusion follows.

II. Uneven Power Distribution Across Subcarriers in MC-CDMA System

The transmitted signal of a downlink MC-CDMA system can be described as

$$s(t) = Re\left\{\sum_{k=0}^{K-1} b^{(k)} A \sum_{n=0}^{N-1} \beta_n^{(k)} e^{j2\pi f_n t} p(t)\right\} \tag{1}$$

where K is the total number of active users, $b^{(k)}$ is the k^{th} user's data symbol ($b^{(k)} \in \{1,-1\}$ if BPSK modulation is used), $A = \sqrt{\frac{2E_s}{N \cdot T_s}}$ is the amplitude, E_s is the symbol energy, T_s is the symbol duration, N is the number of subcarriers, $\beta_n^{(k)}$ is the n^{th} component of user k's spreading code, f_n is the frequency of the n^{th} subcarrier and $f_n = f_c + n \cdot \Delta f$ (where f_c is the carrier frequency and $\Delta f = \frac{1}{T_s}$ to ensure orthogonality among all subcarriers, and $p(t)$ is the rectangular pulse shape.

Normally, a binary code matrix $\mathbf{C}$ is employed to assign spreading codes to all users. The most commonly used spreading code is the Hadamard-Walsh code. In the code matrix $\mathbf{C}$, every row represents a spreading code for one user:

$$\mathbf{C} = \begin{bmatrix} \vec{\beta}^{(0)} \\ \vec{\beta}^{(1)} \\ \vdots \\ \vec{\beta}^{(N-1)} \end{bmatrix} = \begin{bmatrix} \beta_0^{(0)} & \beta_1^{(0)} & \cdots & \beta_{N-1}^{(0)} \\ \beta_0^{(1)} & \beta_1^{(1)} & \cdots & \beta_{N-1}^{(1)} \\ \vdots & & & \\ \beta_0^{(N-1)} & \beta_1^{(N-1)} & \cdots & \beta_{N-1}^{(N-1)} \end{bmatrix} \quad (2)$$

where $\vec{\beta}^{(k)}$ represents the spreading code of the k^{th} user and $\vec{\beta}^{(k)} = (\beta_k^{(0)} \beta_k^{(1)} \ldots \beta_k^{(N-1)})$.

It is important to note that the code matrix is binary, i.e., $\beta_n^{(k)} \in \{1, -1\}$. Hence, oftentimes the transmitted signal has uneven power distribution over all subcarriers. Particularly, what is most problematic is when some subcarriers have zero transmission power.

Let's use an example to demonstrate this. Assume a MC-CDMA system with $N = 8$ subcarriers. A length 8 Hadamard-Walsh code matrix is used as the spreading code matrix. The K active users will randomly pick K rows from this matrix as their spreading codes.

Assume there are two users on the system and one user (the 0^{th} user) is using spreading code $\{1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\}$, while the other user (the 1^{th} user) is using spreading code $\{1\ 1\ 1\ 1\ -1\ -1\ -1\ -1\}$. When both users transmit the same data symbol (for example, $b^{(0)} = 1$ and $b^{(1)} = 1$), the binary nature of the spreading codes lead to transmitted signal over the 8 subcarriers as $\{2\ 2\ 2\ 2\ 0\ 0\ 0\ 0\ \}$. Hence, only the first 4 subcarriers have power and the other 4 subcarriers are actually not transmitting anything. This leads to less frequency diversity: only half of the diversity is exploited in this case. This scenario is shown in Figure 1.

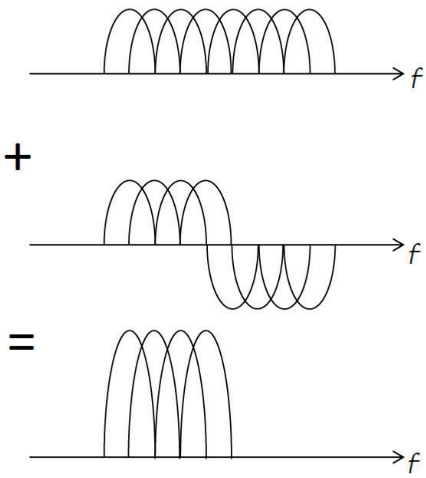

Fig. 1. Uneven Subcarrier Power Distribution 1

If both users are transmitting the opposite data symbols (for example, $b^{(0)} = 1$ and $b^{(1)} = -1$), the actual transmitted signal over the 8 subcarriers becomes $\{0\ 0\ 0\ 0\ 2\ 2\ 2\ 2\}$. Now, only the last four subcarriers are transmitting power while the first four subcarriers have zero power. Therefore, only half of the frequency diversity is exploited.

There are even cases that multiple users' signal accumulate to only one subcarrier containing all the transmission power and all other subcarriers transmitting zero power. For example, if all users transmit data symbol 1, transmission power will be only on subcarrier 0, while all other subcarriers have zero power. In these cases, no diversity is exploited at all.

III. Phase Rotated Code Design

To solve this problem and bring full diversity to MC-CDMA system at all times, we employ the phase rotated spreading code design developed by [8]. Particularly, by rotating each row of the spreading code matrix with a different phase, a new spreading code matrix is created that maintains the orthogonality among all rows. However, this new spreading code matrix eliminates the possibility of zero power accumulation on any subcarrier. Therefore, all subcarriers are actively participating in the demodulation of the data symbol at receiver side, exploiting full frequency diversity available in the frequency selective fading channel at all times.

Figure 2 explains the problem of zero power accumulation. Since user k is transmitting the product of data symbol and spreading code $b^k \beta_n^k$ on the n^{th} subcarrier, user l is transmitting $b^l \beta_n^l$, due to the binary nature of the code β_n^k (and β_n^l) and the data symbol b^k (and b^l, the code/data combination $b^k \beta_n^k$ is either $+1$ or -1. Therefore, it is inevitable that sometimes one user's code/data combination will be $+1$ and another user's code/data combination will be -1 and when they transmit the signal results in a zero, as shown in Figure 2.

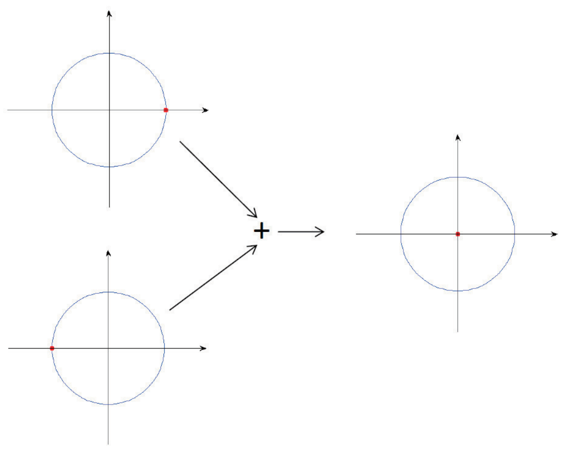

Fig. 2. Binary Code/Data Combination

A new spreading code matrix $\mathbf{C_{New}}$ can be created by introducing an unique phase offset to each and every row of the original Hadamard-Walsh code matrix:

$$\mathbf{C_{New}} = \begin{bmatrix} P^{(0)} \cdot \vec{\beta}^{(0)} \\ P^{(1)} \cdot \vec{\beta}^{(1)} \\ \vdots \\ P^{(N-1)} \cdot \vec{\beta}^{(N-1)} \end{bmatrix} = \begin{bmatrix} e^{j\frac{\pi}{N}0} \cdot \vec{\beta}^{(0)} \\ e^{j\frac{\pi}{N}1} \cdot \vec{\beta}^{(1)} \\ \vdots \\ e^{j\frac{\pi}{N}(N-1)} \cdot \vec{\beta}^{(N-1)} \end{bmatrix} \quad (3)$$

As shown in equation (3), a phase rotator $P^{(k)}$ is multiplied to the k^{th} row of the original Hadamard-Walsh code matrix where $P^{(k)} = e^{j\frac{\pi}{N}k}$. Consequently, each row is rotated by a different amount in the phase space. It is easy to show that the introduction of the phase rotation does not change the orthogonality of the spreading code matrix, i.e., $\mathbf{C_{New}}$ is still an orthogonal matrix. The inner product of the k^{th} row and the l^{th} row of the new code matrix $\mathbf{C_{New}}$ is:

$$< P^{(k)} \cdot \vec{\beta}^{(k)}, P^{(l)} \cdot \vec{\beta}^{(l)} >= P^{(k)} \cdot P^{*(l)} < \vec{\beta}^{(k)}, \vec{\beta}^{(l)} > \quad (4)$$

Since $\mathbf{C}$ is an orthogonal matrix, $< \vec{\beta}^{(k)}, \vec{\beta}^{(l)} >= 0, \forall k \neq l$. Therefore, $< P^{(k)} \cdot \vec{\beta}^{(k)}, P^{(l)} \cdot \vec{\beta}^{(l)} >$ is also 0 for any two different rows in $\mathbf{C_{New}}$.

Therefore, no matter what the code/data combination of every user is, it is guaranteed that they will not accumulate to zero. This is shown in Figure 3. In Figure 3, 8 different phases separated by $\frac{\pi}{8}$ are introduced to the 8 different users' spreading codes. Therefore, if the original binary code/data combination is $+1$ for one user, it will pick one of the 8 different points shown in the up-left constellation; otherwise it will choose one of the 8 points in the bottom constellation. However, it is guaranteed that the sum of two different user's signal will not be zero.

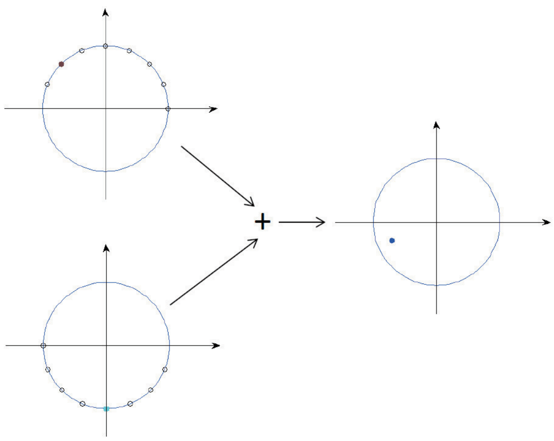

Fig. 3. Phase Rotated Code/Data Combination

IV. Combining Scheme for Phase Rotated Codes

Figure 4 shows the generic block diagram of the MC-CDMA l^{th} user's receiver. Note that in the despreading stage we use $\beta_n^{*(l)}$ (the complex conjugate of $\beta_n^{(l)}$) since we now have a complex spreading code matrix instead of a real matrix.

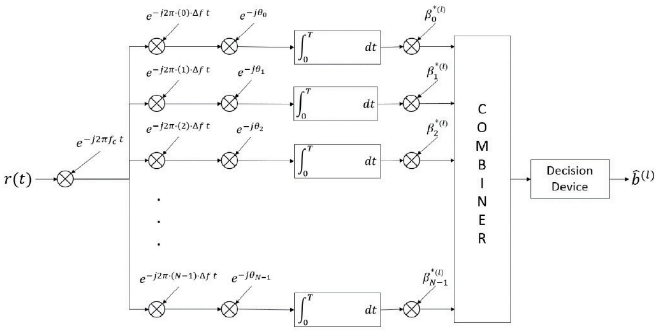

Fig. 4. MC-CDMA Receiver

After the despreading, the n^{th} subcarrier's output is:

$$r_n^{(l)} = A\alpha_n \cdot b^{(l)} + A\alpha_n \sum_{k=0,k\neq l}^{K-1} b^k Re[\beta_n^{(k)}\beta_n^{*(l)}] + n_n \quad (5)$$

where α_n is the fading gain of the n^{th} subcarrier. In equation (5), the first term represents the desired signal of the l^{th} user, the second term represents the multiple access interference (MAI) from the other $K-1$ users, and the third term represents additive Gaussian noise. Next, a linear combiner combines across all subcarriers to form a decision variable:

$$R^{(l)} = W_n \cdot r_n^{(l)} \quad (6)$$

where W_n is the combining weight for the n^{th} subcarrier.

Since we have assumed BPSK modulation, the MAI in equation (5) only contains the real part of the product of $\beta_n^{(k)}$ and $\beta_n^{*(l)}$. Since $\beta_n^{(k)} \in \{e^{j\frac{\pi}{N}k}, -e^{j\frac{\pi}{N}k}\}$ and $\beta_n^{(l)} \in \{e^{j\frac{\pi}{N}l}, -e^{j\frac{\pi}{N}l}\}$, $Re[\beta_n^{(k)}\beta_n^{*(l)}]$ becomes:

$$Re[\beta_n^{(k)}\beta_n^{*(l)}] = \cos\left(\frac{\pi}{N}(k-l)\right) \quad (7)$$

or

$$Re[\beta_n^{(k)}\beta_n^{*(l)}] = \cos\left(\frac{\pi}{N}(k+l)\right). \quad (8)$$

Without losing generality, assume $l = 0$, we can easily derive the minimized mean square error combining (MMSEC) scheme to be:

$$W_n = \frac{\alpha_n}{KA^2\alpha_n^2/2 + \frac{N_0}{2}} \quad (9)$$

where $\frac{N_0}{2}$ is the power spectral density of the AWGN.

It is important to note that because of the phase rotation, the MAI observed at the MC-CDMA receiver using the phase rotated codes (with BPSK modulation) at each subcarrier is only half of that when binary Hadamard-Walsh codes are used. Hence, when BPSK modulation is employed, the phaser rotation spreading code design provides a two-fold benefit: on one hand, the MAI power is reduced in half; on the other hand, full diversity is always exploited. Unfortunately, the MAI power reduction benefit disappears when high modulations such as QPSK and QAM are employed.

Now let's revisit previously developed CI/MC-CDMA system [10]. In CI/MC-CDMA system, the spreading code matrix is the DFT matrix. The k^{th} user's spreading code is:

$$\vec{\beta}^{(k)} = \left(e^{-j\frac{2\pi}{N}\cdot k\cdot 0}, e^{-j\frac{2\pi}{N}\cdot k\cdot 1}, \ldots, e^{-j\frac{2\pi}{N}\cdot k\cdot (N-1)}\right) \quad (10)$$

It has been shown in [10] that CI/MC-CDMA system outperforms MC-CDMA system employing Hadamard-Walsh codes when BPSK modulation is employed. However, when higher modulation such as QPSK is employed, CI/MC-CDMA no longer offers such performance gain over MC-CDMA with Hadamard-Walsh codes.

Now we can provide an explanation of this distinction. On subcarrier n, i.e., the n^{th} chip of the k^{th} user's spreading code is $\beta_n^{(k)} = e^{-j\frac{2\pi}{N}\cdot k\cdot n}$. When $n = 0$, $\beta_n^{(k)} = e^{-j\frac{2\pi}{N}\cdot k\cdot 0} = 1$. When $n = \frac{N}{2}$, $\beta_n^{(k)} = e^{-j\frac{2\pi}{N}\cdot k\cdot \frac{N}{2}} = e^{-jk\pi} \in \{+1, -1\}$. In other words, on these two subcarriers, all spreading codes are binary. However, on other subcarriers (i.e.,$n \neq 0, n \neq \frac{N}{2}$, the spreading codes from different users are actually phase rotated. As a direct result, CI/MC-CDMA enjoys the same MAI reduction benefit as the phase rotated MC-CDMA system on $N-2$ subcarriers out of the total N subcarriers. This is the source of the performance gain of CI/MC-CDMA shown in [10].

V. Numerical Results

In this section, we simulate the BER performance of the proposed code design for MC-CDMA system under different conditions to validate its benefits. To model realistic wireless environments, the Rayleigh fading channel employed in our simulation demonstrates frequency selectivity over the entire bandwidth, BW, but flat fading over each of the N carriers. Specifically, we assumed a channel model with coherence bandwidth, $(\Delta f)_c$, characterized by $(\Delta f)_c/BW = 0.25$. As a result, the fading gains in the N carriers are correlated according to

$$\rho_{i,j} = \frac{1}{1 + ((f_i - f_j)/(\Delta f)_c)^2} \quad (11)$$

where $\rho_{i,j}$ denotes the correlation between the i^{th} carrier and the j^{th} carrier, and $(f_i - f_j)$ is the frequency separation between these two carriers. Generation of correlated fades, for purposes of simulation, has been discussed in [11].

Now, we simulate MC-CDMA systems with $N = 32$ subcarriers. As a benchmark, a MC-CDMA system using length 32 Hadamard-Walsh codes and BPSK modulation is simulated first. Then a MC-CDMA system using our proposed phase rotated Hadamard-Walsh length 32 codes and BPSK modulation is simulated. Figure 5 illustrates the bit error rate (BER) versus average signal to noise ratio (SNR) performances for a fully loaded system, i.e., $K = N = 32$. Figure 6 shows the BER vs SNR performances for a half loaded system, i.e., $K = \frac{N}{2} = 16$, and Figure 7 illustrates the same curves for a quarterly loaded system ($K = 8$). In all three figures, the line marked with circles represents the performance of conventional Hadamard-Walsh codes, while the line marked with stars represents that of our proposed phase rotated codes. It is evident from all figures that the proposed new codes significantly outperform the conventional Hadamard-Walsh codes. At $BER = 10^{-4}$, the new codes provide approximately 2dB gain over the Hadamard-Walsh codes.

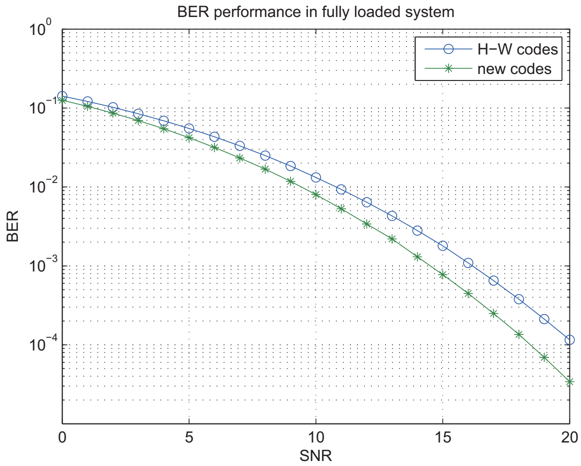

Fig. 5. BER vs SNR, K=32

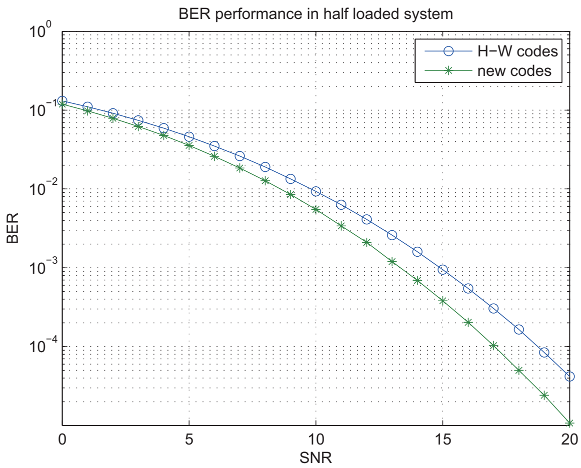

Fig. 6. BER vs SNR, K=16

Next, we show the BER performance versus number of active users K on the MC-CDMA system at different average SNRs. Figure 8 illustrates the BER vs number of users for both Hadamard-Walsh codes and the new codes at SNR=18dB, while Figure 9 shows the case of SNR=10dB. In both figures, the line marked with circles represents the performance of conventional Hadamard-Walsh codes, while the line marked with stars represents that of our proposed phase rotated codes. Notice that when $K = 1$, both the new codes and the Hadamard-Walsh codes provide exactly the same performance. This is because when there is only one user on the system,

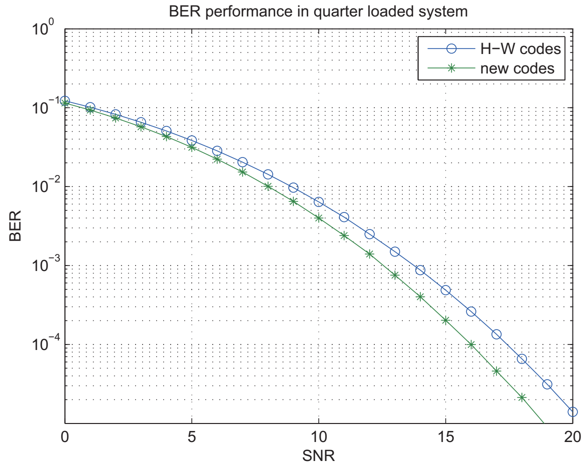

Fig. 7. BER vs SNR, K=8

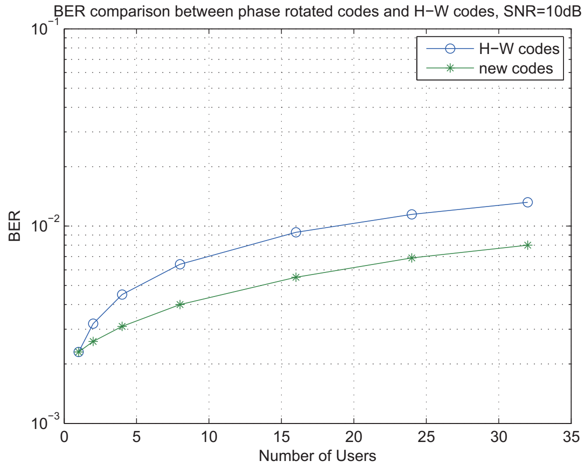

Fig. 9. BER vs K, SNR=10dB

both codes spread information over all subcarriers and full frequency diversity is exploited. When the number of users K increases, the BER worsens due to the increased amount of multiple access interference (MAI). However, the proposed phase rotated codes provide much better performance than its Hadamard-Walsh counterpart. This is due to the fact that the new codes provide less MAI and full diversity. It is evident from all figures that the fully loaded system employing the new codes offers better BER performance than that of a half loaded system employing Hadamard-Walsh codes. The gain is clearly significant.

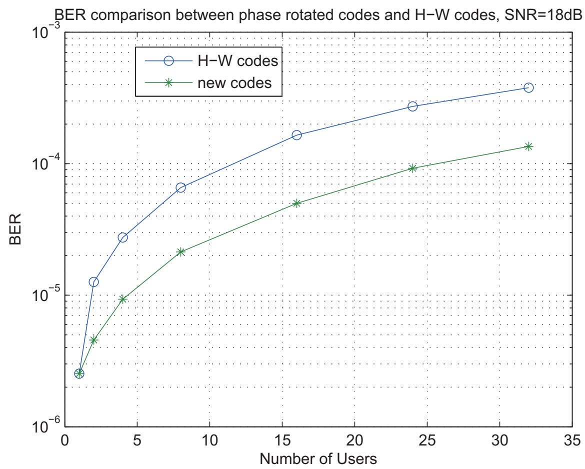

Fig. 8. BER vs K, SNR=18dB

VI. Conclusion

In this paper, we have designed a low complexity minimized mean square error combining scheme for downlink MC-CDMA systems using phase rotated spreading codes. The proposed scheme eliminates the problem of zero power distribution on subcarriers. As a direct result, full diversity is always exploited and significant performance gain is achieved in multi-path fading channels. Simulations over various channel conditions and scenarios confirm the effectiveness of the proposed scheme.

Acknowledgment

This material is based upon work supported by the National Science Foundation under Grant No. 1323240, and the Office of Naval Research. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the funding agencies.

References

[1] S. Hara and R. Prasad, "Overview of multi-carrier CDMA", *IEEE Communications Magazine*, vol. 35, no. 12, Dec. 1997, pp. 126-133

[2] L. Rugini, "Linear Equalization for Multicode MC-CDMA Downlink Channels," *IEEE Communications Letters*, vol.16, no.9, pp.1353-1356, September 2012.

[3] T. Miyajima, and M. Kotake, "Blind Channel Shortening for MC-CDMA Systems by Restoring the Orthogonality of Spreading Codes," *IEEE Transactions on Communications*, vol.63, no.3, pp.938-948, March 2015.

[4] R. Rajbanshi, Q. Chen, A. M. Wyglinski, G. J. Minden and J. B. Evans, "Quantitative Comparison of Agile Modulation Techniques for Cognitive Radio Transceivers", *IEEE CCNC2007, First Workshop on Cognitive Radio*, Las Vegas, January, 2007.

[5] Z. Wu and C. R. Nassar, "FD-MC-CDMA: A Frequency-based Multiple Access Architecture for High Performance Wireless Communication," *IEEE Transactions on Vehicular Technology*, vol. 54, no. 4, pp. 1392-1399, July 2005

[6] S. Hijazi, B. Natarajan, M. Michelini and Z. Wu, "Flexible Spectrum Use and Better Coexistence at the Physical Layer of Future Wireless Systems via a Multicarrier Platform", *IEEE Wireless Communications*, April 2004, Vol.11, No. 2, pp. 64-71.

[7] Z. Wu, B. Natarajan and C. Nassar, "The Road to 4G: Two Paradigm Shifts, One Enabling Technology", *IEEE DySPAN2005*, 2005.

[8] A. Bury, J. Egle and Jrgen Lindner, "Diversity Comparison of Spreading Transforms for Multicarrier Spread Spectrum Transmission," *IEEE Transactions on Communications*, Vol. 51, No. 5, pp. 774-781, May 2003

[9] R. Raulefs, A. Dammann, S. Sand, S. Kaiser and Gunther Auer, "Rotated Walsh-Hadamard Spreading with Robust Channel Estimation for a Coded MC-CDMA System," *EURASIP Journal on Wireless Communications and Networking*, 2004:1, 7483

[10] B. Natarajan, C.R. Nassar, S. Shattil and Zhiqiang Wu, "High-Performance MC-CDMA via Carrier Interferometry Codes," *IEEE Transactions on Vehicular Technology*, vol. 50, no. 6, pp. 1344-1353, November 2001

[11] B. Natarajan, C.R. Nassar and V. Chandrasekhar, "Generation of Correlated Rayleigh Fading envelops for spread spectrum applications", *IEEE Communication Letters*, vol. 4. no.1. Jan, 2000, pp. 9-11.