

Mathematical Modelling of an 3 Phase Induction Motor Using MATLAB/Simulink

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ABSTRACT

Mechanical energy is needed in the daily life use as well as in the industry. Induction motors play a very important role in both worlds, because of low cost, reliable operation, robust operation and low maintenance. To derive the mathematical model of a 3 phase Induction motor, the theory of reference frames has been effectively used as an efficient approach. Dynamic models (mathematical models) are employed in to better understand the behaviour of induction motor in both transient and steady state. The dynamic modelling sets all the mechanical equations for the inertia, torque and speed versus time. It also models all the differential voltage, currents and flux linkages between the stationary stator as well as the moving rotor. This paper presents a step by step Matlab/Simulink implementation of an induction machine using dq0 axis transformations of the stator and rotor variables in the arbitrary reference frame [1].

Keywords: Reference frames, Dynamic models, dq0 axis transformations, Matlab/Simulink.

I. INTRODUCTION

The dynamic simulation is one of the key steps in the validation of the design process of the motor-drive system, which eliminates the designing mistakes and the resulting errors in the prototype construction and testing. The dynamic model of the induction motor in direct, quadrature, and zero-sequence axes can be derived from fundamental equations of transformation. The dynamic analysis of the symmetrical induction machines in the arbitrary reference frame has been intensively used as a standard simulation approach from which any particular mode of operation may then be developed. Matlab/Simulink has an advantage over other machine simulators in modelling the induction machine using dq0 axis transformation. Generally modelling of these equations is considered difficult so that in this paper they are presented in their simplified form. The transformations used at various steps are based on simple trigonometric relationship obtained as projections on a set of axes. The dynamic model is used to obtain transient responses, small signal equations, and a transfer function of induction motor. Dynamic models (mathematical models) are employed in to better

understand the behaviour of induction motor in both transient and steady state. The dynamic modelling sets all the mechanical equations for the inertia, torque and speed versus time. It also models all the differential voltage, currents and flux linkages between the stationary stator as well as the moving rotor. This mathematical model has been done by using MATLAB /Simulink which will represent the three phase induction motor including a three phase to d-q axis transformations. The main benefit with MATLAB Simulink is that in the electromechanical dynamic model can be accomplished in a simple way and it can be simulated faster using function blocks.

II. METHODS AND MATERIAL

1. Mathematical Modeling of 3-Phase Induction Motor

The model constructed according to the equations has been simulated by using MATLAB/SIMULINK. A 3 phase source is applied to conventional model of an induction motor and the equations are given by:

$$V_a = \sqrt{2}V_{rms} \sin wt \dots\dots\dots (1)$$

$$V_b = \sqrt{2}V_{rms} \sin(wt - 2\pi/3) \dots\dots\dots (2)$$

$$V_c = \sqrt{2}V_{rms} \sin(wt + 2\pi/3) \dots\dots\dots (3)$$

These three-phase voltages are transferred to a synchronously rotating reference frame in only two phases (d-q axis transformation). This can be done using the following equations.

$$\begin{pmatrix} V_\alpha \\ V_\beta \\ V_o \end{pmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} \dots\dots\dots (4)$$

Then, the direct and quadrature axes voltages are:

$$\begin{pmatrix} V_d \\ V_q \\ V_o \end{pmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} V_\alpha \\ V_\beta \\ V_o \end{pmatrix} \dots\dots\dots (5)$$

For a 2-phase machine, we need to represent both *ds-q*s and *dr-qr* circuits and their variables in a synchronously rotating *de-qe* frame. The stator circuit equations can be modeled as follows:

$$V_{qs}^s = R_s I_{qs}^s + \frac{d\phi_{qs}^s}{dt} \dots\dots\dots (6)$$

$$V_{ds}^s = R_s I_{ds}^s + \frac{d\phi_{ds}^s}{dt} \dots\dots\dots (7)$$

where ϕ_{qs}^s and ϕ_{ds}^s are the *q*-axis and *d*-axis stator flux linkages, respectively. Equations (6) and (7) are further converted into *de-qe* frames as

$$V_{qs} = R_s I_{qs} + \frac{d\phi_{qs}}{dt} + We \cdot \phi_{ds} \dots\dots\dots (8)$$

$$V_{ds} = R_s I_{ds} + \frac{d\phi_{ds}}{dt} - We \cdot \phi_{qs} \dots\dots\dots (9)$$

Note that in the above equations, all the variables are in rotating form. The last terms in equations (8) and (9) are the speed emfs due to the rotation of the axes, i.e., when $\omega_e = 0$, the equations revert back to the stationary form. The flux linkages in the *de* and *qe* axes induce emfs in the *qe* and *de* axes with a leading angle of 90. If the rotor is not moving, i.e., $\omega_r = 0$, then the rotor equations for a

doubly fed wound rotor IM will be similar to the stator equations and are given by

$$V_{qr} = R_r I_{qr} + \frac{d\phi_{qr}}{dt} + We \cdot \phi_{dr} \dots\dots\dots (10)$$

$$V_{dr} = R_r I_{dr} + \frac{d\phi_{dr}}{dt} - We \cdot \phi_{qr} \dots\dots\dots (11)$$

where all the variables and parameters are referred to the stator. Since the rotor moves at a speed of $\omega_r = \theta$, the *d-q* axes fixed on the rotor moves at a speed of $\omega_e - \omega_r$ relative to the synchronously rotating frame. Therefore, in *de-qe* frame, the rotor equations can be rewritten as follows:

$$V_{qr} = R_r I_{qr} + \frac{d\phi_{dr}}{dt} + (We - Wr)\phi_{dr} \dots\dots\dots (12)$$

$$V_{dr} = R_r I_{dr} + \frac{d\phi_{dr}}{dt} - (We - Wr)\phi_{qr} \dots\dots\dots (13)$$

The flux linkage expressions in terms of the currents can be written similar to that in equations (14)–(19). Using equations (14)–(19) in equations (8)–(13), the electrical transient model of the IM in terms of *v* and *i* is given in matrix form as in Equation (20); in the inverse form, the *i-v* matrix could be written as in Equation (21).

$$\phi_{qs} = L_{ls} I_{qs} + L_m(I_{qs} + I_{qr}) \dots\dots\dots (14)$$

$$\phi_{qr} = L_{lr} I_{qr} + L_m(I_{qs} + I_{qr}) \dots\dots\dots (15)$$

$$\phi_{qm} = L_m(I_{qs} + I_{qr}) \dots\dots\dots (16)$$

$$\phi_{ds} = L_{ls} I_{ds} + L_m(I_{ds} + I_{dr}) \dots\dots\dots (17)$$

$$\phi_{dr} = L_{lr} I_{dr} + L_m(I_{ds} + I_{dr}) \dots\dots\dots (18)$$

$$\phi_{dm} = L_m(I_{ds} + I_{dr}) \dots\dots\dots (19)$$

$$\begin{pmatrix} V_{qs} \\ V_{ds} \\ V_{qr} \\ V_{dr} \end{pmatrix} = \begin{bmatrix} R_s + s L_s & -We L_s & s L_m & -We L_m \\ -We L_s & R_s + s L_s & -We L_m & s L_m \\ s L_m & (We - Wr) L_m & R_r + s L_r & (We - Wr) L_r \\ -(We - Wr) L_m & s L_m & -(We - Wr) L_r & R_r + s L_r \end{bmatrix} \begin{pmatrix} I_{qs} \\ I_{ds} \\ I_{qr} \\ I_{dr} \end{pmatrix} \dots\dots (20)$$

$$\begin{pmatrix} I_{qs} \\ I_{ds} \\ I_{qr} \\ I_{dr} \end{pmatrix} = \begin{bmatrix} R_s + s L_s & -We L_s & s L_m & -We L_m \\ -We L_s & R_s + s L_s & -We L_m & s L_m \\ s L_m & (We - Wr) L_m & R_r + s L_r & (We - Wr) L_r \\ -(We - Wr) L_m & s L_m & -(We - Wr) L_r & R_r + s L_r \end{bmatrix}^{-1} \begin{pmatrix} V_{qs} \\ V_{ds} \\ V_{qr} \\ V_{dr} \end{pmatrix} \dots\dots (21)$$

where 's' is the Laplacian operator. For a single phase IM, such as the cage motor, $v_{qr} = v_{ds} = 0$. If the speed ω_r is considered to be constant (infinite inertia load), the

III. RESULTS AND DISCUSSION

MATLAB/SIMULINK RESULTS

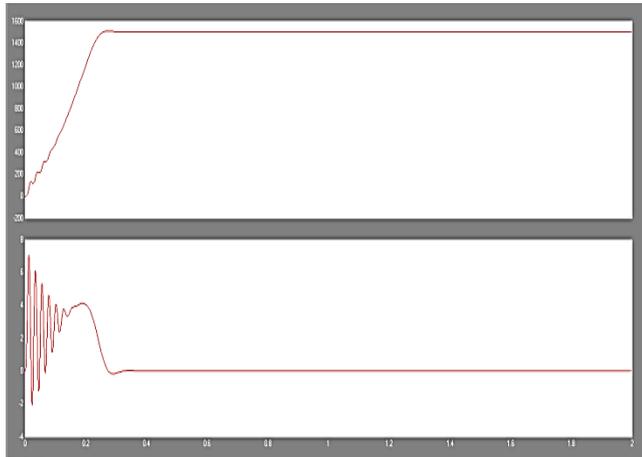


Figure 2. Speed and Torque of I.M

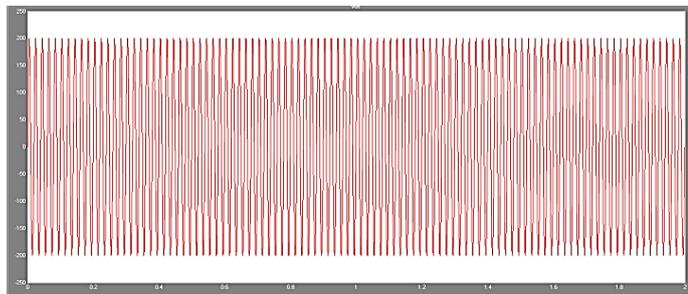


Figure 4. V_{ds}

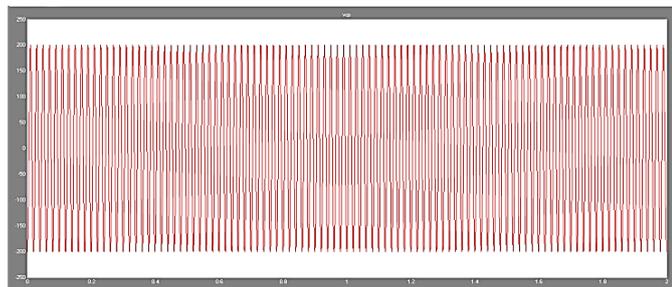


Figure 5. V_{qs}

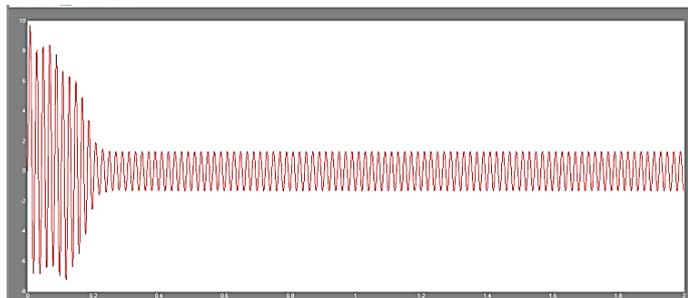


Figure 6. i_{ds}

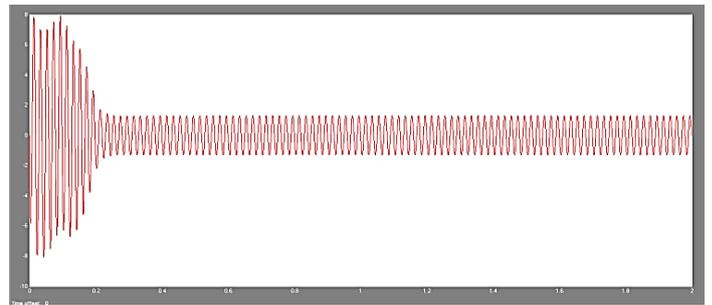


Figure 7. i_{qs}



Figure 8. i_{dr}



Figure 9. i_{qr}

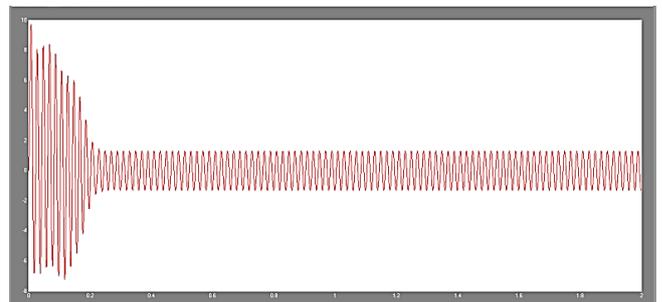


Figure 10. i_{as}

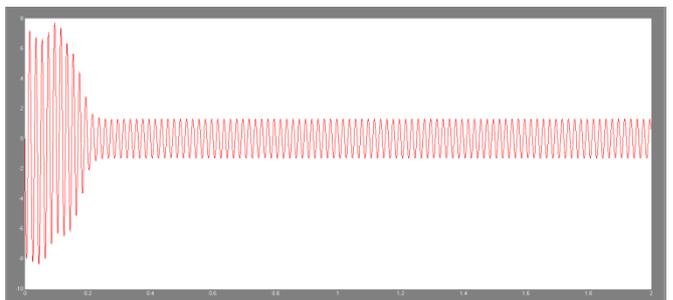


Figure 11. i_{bs}

IV. CONCLUSION

In this paper, an implementation and dynamic modelling of a three-phase induction motor using Matlab/Simulink is presented in a step-by-step manner. The model was tested. The simulated machine has given a satisfactory response in terms of the torque and speed characteristics. This concludes that the Matlab/Simulink is a reliable and sophisticated way to analyse and predict the behaviour of induction motors using the theory of reference frames.

V. REFERENCES

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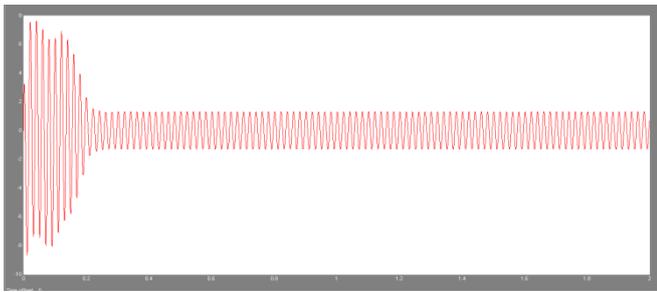


Figure 12. ics



Figure 13. iar



Figure 14. ibr



Figure 15. icr

```
% Parameters of a typical induction
machine.
rs=6.03; %Stator resistance
rr=6.085; %Rotor resistance
Ls=489.3e-3;%Stator inductance
Lr=489.3e-3;%Rotor inductance
M=450.3e-3; % Mutual inductance
P=4; %Poles
J=0.00488;%Inertia
sigma=(Ls*Lr) - (M*M);
```