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The application of internal model fractional order control in induction motor speed control system

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Abstract: According to the principle of IMC, a fractional-order internal model controller which is exchanged integer order filter for fractional order filter is deduced for induction motor speed control system. This controller contains two adjustable parameters, and the controller parameter setting method is given according to the phase margin method. Then, the algorithm is applied to the experimental platform of induction motor speed control system which is based on TMS320LF2407 DSP. The simulating and experimental results show that the induction motor speed control system can obtained good following performance, nice anti-interference performance and stronger robustness with the controller.

Keywords: Induction motor, IMC, Fractional order filter, Phase margin method, DSP

1 Introduction

With the development of the theory of fractional calculus, fractional-order control theory has been received widespread attention in the control field and has been applied to motion control system by scholars in recent years. Fractional-order controller $PD^{\mu}$and $[PD]^{\mu}$ which are used in the servo system are respectively designed according to the references[1-2]. According to the paper[3], the fuzzy fractional order sliding-mode control method is applied to the servo system of permanent magnet synchronous motor, and the simulation results show that this control method not only effectively weakens the chattering phenomenon in traditional sliding mode control system but also improves the robustness of the system. Comparing with integer order controller, the parameters of calculus order is increased in fractional order controller [4]. So the integrated parameter is difficult. To solve this problem, some scholars put forward a fractional order controller which is based on internal model structure. In other words, based on the principle of internal IMC, a fractional order controller $PI^{\lambda}D^{\mu}$ which is used to an integer order system is derived by introducing fractional order filter. This controller contains two adjustable parameters, which not only overcomes the hard to set the parameters of fractional-order controller but also can improve the control performance of the system.

In the induction motor speed control system, it is very important to implement the transformation from theory research to engineering application. At present, the performance of control chips with operating all kinds of control algorithms is constantly improved in practical engineering. DSP (digital signal processor) chip which is used to optimize the structure of the control has been widely applied to the field of control due to its fast computing speed, ability to processors and rich peripherals resources. In this article, the effect of the designed controller is testified by doing experiments in the platform which are based on DSP control chip TMS32LF2407.

2 Mathematical model of induction motor speed control system

Induction motor with rotor flux oriented vector control scheme [5] is shown in fig. 1. A can be seen, the scheme is based on the measurement of two phase's currents and of the motor position. The Slip compensation block is used to estimate the stator field position.

Where ω_r^* and ω_r are respectively the given speed and the actual speed, respectively. i_q^* and i_d^* are the given input of current controller of q and d axis, respectively. u_q^* and u_d^* are the output of current controller of q and d axis, respectively. u_a^* , u_b^* and u_c^* are the given signal of three-phase voltage which were obtained by the coordinate transformation[7], and are also the input of PWM (pulse width modulation) pulse generator. The output of PWM is 6-line pulse signal, which can drive inverter to control the induction motor. i_a, i_b and i_c are the output of the inverter. i_α and i_β are the results of the Clarke transformation[7]. i_d and i_q are the results of the Park transformation[7]. θ is the stator magnetic field Angle.

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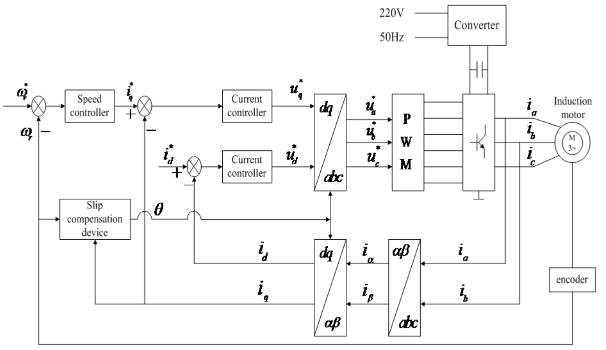

Fig. 1 Induction motor system based on vector control.

Known from Fig. 1, the d axis is the rotor magnetic field direction, which is perpendicular to q axis. The flux of rotor is zero in q axis. That is, $\psi_q=0$. Therefore, the induction motor torque equation is:

$$T_e = \frac{3}{2} n_p \frac{L_m}{L_r}(i_q\psi_d - i_d\psi_q) \tag{1}$$

Which can be written as

$$\begin{aligned} T_e &= \frac{3}{2}n_p\frac{L_m}{L_r}i_q\psi_d = \frac{3}{2}n_p\frac{L_m}{L_r}i_q(L_m i_d) \\ &= \frac{3}{2}n_p\frac{L_m^2}{L_r}i_d i_q = K_t i_q \end{aligned} \tag{2}$$

Where T_e is electromagnetic torque, n_p is logarithmic machine, L_m is the mutual inductance between the stator and rotor and L_r is the winding inductance of rotor. $\psi_d = L_m \cdot i_d$ is the flux linkage of rotor. ψ_q is the flux linkage of stator. i_d and i_q are the current of q and d axis, respectively. $K_t = (3/2)n_p(L_m^2/L_r)i_d$ represents torque constant.

According to Eq. (2) and Fig.1, id is a fixed numerical value. As long as iq is controlled, the instantaneous value of electromagnetic torque will be controlled. So the rotor flux orientation vector control of induction motor can be realized.

Equations of induction motor is:

$$T_e - T_L = J\frac{d\omega}{dt} + B\omega \tag{3}$$

Where ω is rotor velocity, T_L is load torque, J is mechanical inertia constant and B is the viscous coefficient.

Combining Eq.(1) with Eq. (2), the mathematical model of induction motor speed control system can be obtained. The corresponding simplified block diagram is shown in Fig. 2. G_c(s) is the speed controller.

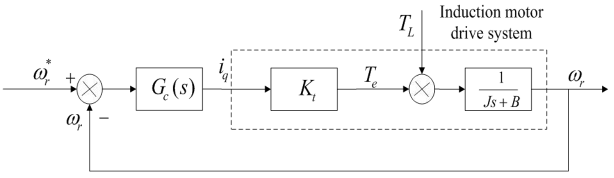

Fig. 2 Simplified induction motor drive system.

The fig. 2 shows that the controlled object of induction motor speed control system is:

$$G(s) = \frac{K_t}{Js+B} = \frac{K_t b}{s+a} = \frac{K}{Ts+1} \tag{4}$$

Where s is the Laplace operator.

3 The design of fractional order controllers

Based on IMC principle, when induction motor speed control system is No-load, namely, the load torque $T_L = 0$, the speed controller can be designed, and the feedback controller can be obtained by equivalent transformation. The structure diagram of speed controller is shown in Fig. 3.

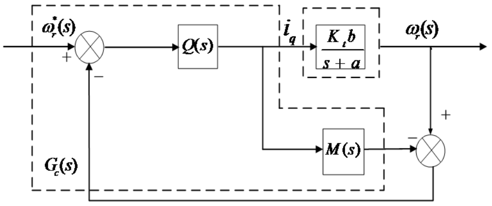

Fig. 3 IMC structure

Where M(s) is the internal model and Q(s) is the internal model controller.

The block diagram for the conventional feedback control is given in Fig. 4. G_c(s) is the equivalent of feedback controller. It can be shown that the feedback controller G_c(s) and the internal model controller Q(s) has the following relation.

$$G_c(s) = \frac{Q(s)}{1 - Q(s)M(s)} \tag{5}$$

By the IMC principle, the IMC $Q(s) = M_-^{-1}(s)f(s)$, where $M_-^{-1}(s)$ is the minimum phase of model and $f(s)$ is filter which is different in this article. Selected filter form is:

$$f(s) = \frac{1}{1+\lambda s^\gamma} \qquad 1 < \gamma < 2 \tag{6}$$

Where γ is the order of the filter of fractional-order controller and λ is filtering time constant.

Fractional order IMC speed controller is:

$$Q(s) = \frac{s+a}{k_t b}\cdot\frac{1}{(1+\lambda s^\gamma)} \tag{7}$$

Feedback speed controller is:

$$\begin{aligned} G_c(s) &= \frac{1}{k_t b\lambda s^{\gamma-1}}(1+\frac{a}{s}) \\ &= \frac{1}{k_t b\lambda s^{\gamma-1}} + \frac{a}{k_t b\lambda s^\gamma} \end{aligned} \tag{8}$$

From above-mentioned, internal model fraction-order controller contains two adjustable parameters γ and λ. When $1<\gamma<2$, the controller structure of fractional order controller is II (two integration).

4 Controller parameters setting

Internal model fractional order controller contains two adjustable parameters which increase not only the degree of freedom of the controller but also the complexity of its parameters setting. So the fractional order controller parameters setting is an important research direction in the field of fractional order control. After decades of development, the mature method of fractional order control parameters setting mainly includes: dominant pole method, optimization method and amplitude margin and phase margin. Among them, because the controlled object model is not precise enough, its parameters will change with the variation of working conditions and time because the controlled object model is not precise enough. These factors will make the operation of system deviating from the design requirements. Even more it will make system unstable. So the robustness of the control system should be the primary problem to make sure the system work normally. The stability margin ϕ_m can be used as the major index to measure robustness of system because of the connection between phase and damping of system. So, ϕ_m is often used as an important indicator of setting the controller parameter to make it meet the requirements of the performance of the system. This article uses this phase margin method for setting the parameters of the controller.

The open-loop of induction motor speed control system is:

$$L(s) = G_c(s)G(s) \tag{9}$$

Where $G_c(s)$ is the transfer function of controller and $G(s)$ is the controlled object.

If phase margin and cut-off frequency of system are given, there are three rules for setting controller parameters [8].

Firstly, the angle characteristic which is in the cut-off frequency ω_c of open-loop transfer function of induction motor speed control system is:

$$\phi_m = \pi + Arg[L(j\omega_c)] \tag{10}$$

Secondly, the amplitude property which is in the cut-off frequency ω_c of open-loop transfer function of induction motor speed control system is:

$$|L(j\omega_c)| = |G_c(j\omega_c)G(j\omega_c)| = 1 \tag{11}$$

Thirdly, gain robustness condition of induction motor speed control system.

In order to guarantee the robustness of system due to gain variation, the robustness condition of induction motor speed control system when gain changes should be increased. In other words, the phase Bode diagram of the open-loop transfer function of induction motor speed control system is required to be flat near the cut-off frequency ω_c. That is to say, when the gain of induction motor speed control system is increased (or decreased) by 20%, the derivative of phase to frequency in the cut-off frequency is 0. The overshoot amount of dynamic response of induction motor speed control system remains invariability. Therefore, induction motor speed control system will have a good robustness. The phase of open-loop transfer function of induction motor speed control system meet the following relationship.

$$\left(\frac{d(Arg[G_c(j\omega)G(j\omega)])}{d\omega}\right)_{\omega=\omega_c} = 0 \tag{12}$$

According to E_q.(4), E_q.(8) and E_q.(9), the open loop transfer function is

$$L(s) = \frac{1}{\lambda s^\gamma} \tag{13}$$

Shown in E_q.(13), $\mathrm{Arg}[\mathrm{L}(j\omega_c)] = -\pi\gamma/2$ is a fixed value, so the cut-off frequency phase change rate is 0. Then, There is a horizontal area near the corresponding cut-off frequency of the phase Bode map, which the closed-loop system will have a strong robustness when the gain of closed-loop system changes. According to the E_q.(10) and E_q.(11), the following relationship is:

$$\phi_m = \pi - \frac{\pi}{2}\gamma \tag{14}$$

$$|L(j\omega_c)| = \frac{1}{\lambda\omega_c^\gamma} = 1 \tag{15}$$

Then, γ and λ is deduced. There are:

$$\begin{cases} \gamma = 2 - \dfrac{2}{\pi}\phi_m \\ \lambda = \dfrac{1}{\omega_c^\gamma} \end{cases} \tag{16}$$

Shown in E_q. (16), the initial value of controller parameters can be obtained when the value of phase margin ϕ_m and cut-off frequency ω_c is given.

5 The realization of Controller

Because the transfer function of fractional order control system is infinite dimensional which must use the finite differential method to approach the fractional order control system approximately, the major means of studying fractional-order control system is the discretization and approximation [9].

5.1 Digital implementation

The realization methods of fractional-order controller are mainly analytical method, numerical method and the Z domain numerical method in time domain, of which analytical method is mainly used in the demonstration of theoretical derivation, and Z domain numerical method and the time-domain numerical method are widely used in practical application. Time-domain numerical method is used to achieve the fractional-order controller in this paper.

The fractional-order controller transfer function is:

$$G_c = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu}, \quad \lambda, \mu > 0 \tag{17}$$

The output equation in time domain is:

$$u(t) = K_p e(t) + K_i D^{-\lambda} e(t) + K_d D^{\mu} e(t) \tag{18}$$

The discrete fractional-order controller [10] is:

$$\begin{aligned} u(k) &= K_p e(k) + K_i h^{\lambda} \sum_{j=0}^{k} q_j e(k-j) \\ &+ K_d h^{-\mu} \sum_{j=0}^{k} d_j e(k-j) \end{aligned} \tag{19}$$

Where q_j is the integral operator, d_j is differential operator, $e(k)=r(k)-y(k)$ is the deviation and h is time step.

In approximate calculation, when the numerical approximation method [11] is use to realize the controller, the recursion formula of fractional order differential coefficient is:

$$d_0^{(\alpha)} = 1; \quad d_j^{(\alpha)} = (1 - \frac{\mu+1}{j}) d_{j-1}^{(\alpha)}; \quad j = 1,2,3\cdots \tag{20}$$

Similarly, the recursion formula of fractional integral coefficient is:

$$q_0^{(\alpha)} = 1; \quad q_j^{(\alpha)} = (1 - \frac{\lambda-1}{j}) q_{j-1}^{(\alpha)}; \quad j = 1,2,3\cdots \tag{21}$$

From E$_q$.(20) and E$_q$.(21), as one can see, the differential operator d_j and the integral operator q_j are associated with the current error number and its order. It is unlikely to cumulate error of all points in the practical application. So the thought of "short mnemonic [11, 12]" can deal with it.

In conclusion, when the differential and integral of fractional order controller PI$^{\lambda}$D$^{\mu}$ is approximately replaced by fractional order differential and integral operator, the order of approximate model is higher. Namely, the memory length is much longer, the real system is much closer [11].

5.2 Algorithm process of fractional order controller

First of all, the discrete expression of fractional-order controller PI$^{\lambda}$D$^{\mu}$ should be implemented with the program to achieve fractional-order control algorithm, and its flow chart is shown in Fig. 5.

According to the actual hardware platform, the number of chosen error is k. In consideration of the actual operation time with the complexity of algorithm, the differential and the integral operator can be firstly calculated using iterative formula in initialization program to save running time.

The software flow chart is shown in Fig. 6.

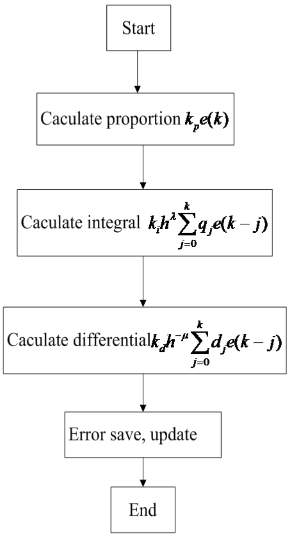

Fig. 5 The flow chart of fractional order control algorithm.

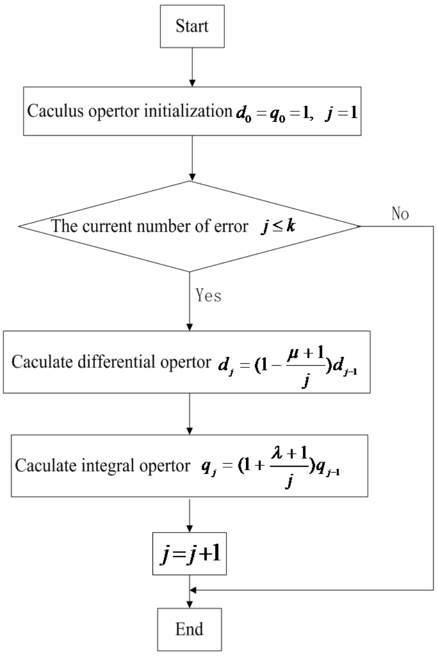

Fig. 6 The flow chart of calculating micro/integral operator of fractional order.

6 The analysis of results

6.1 The result analysis of simulation

By using the MATLAB/SIMULINK simulation tool, the control performance of the proposed method can be testified when using the integral time absolute error (ITAE) and overshoot (σ%) as indicators of the quality of the induction motor of the closed-loop system. The selected induction motor parameters are shown in following Table 1.

Tabal 1 Parameters of the induction motor.

Parameters			
Rated power	0.37 kw	Motor pole	1
Rated speed	2800 r/min	Moment of inertia(J)	0.8182 Kg·m^2
Rated current	1.7A	Moment of gain coefficient(k_t)	0.1898 N·m/A
Rated voltage	230V	Friction drag coefficient(B)	0.0004218 N·m/rad

For fair comparison, cut-off frequency ω_c is chosen 10rad/s and phase margin φ_m is chosen 0.4π (radians). In the simulating, the parameters of the PI$^\alpha$ controller is set by frequency domain criteria according to the paper [13], and the parameters of controller are set by the method proposed in this paper. When the given speed of induction motor speed control system is r(t) =900rpm and the given disturbance torque is $T_L(t\text{-}2)$ =50N · M, the performance index and step response curve of induction motor control system from the two aspect of model matching and model mismatch are shown in Fig. 6 , Fig. 7 and table 2.

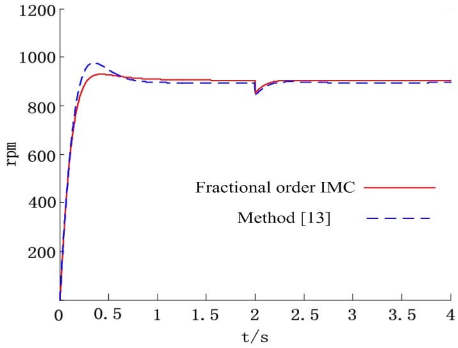

Fig.7 The different speed curves with different setting methods under model matching.

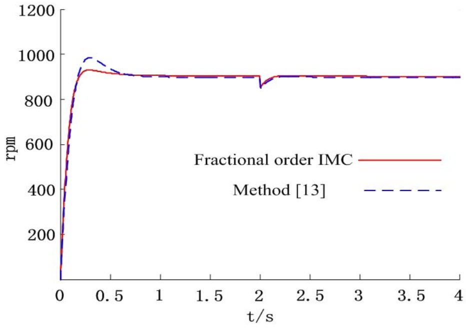

Fig. 8 The different speed curves with different setting methods under mismatch model.

Table 2 Performance parameters of the control system

Methods	Model matching		Model mismatch	
	σ %	ITAE	σ %	ITAE
Method proposed in this paper	3.27	25.81	3.30	21.39
Method proposed in [13]	8.48	60.17	9.60	51.02

The speed response curves are shown in Fig. 7 and Fig. 8. The performance parameters are shown in Table 2. As one can see in Fig. 7 and Table 1, the overshoot amount (σ %) and time multiplied by the absolute error integral (ITAE) are less than those in [13] when the parameters of controller of induction motor speed control system are set by the method of this paper in matching model. It demonstrates the controller parameter setting method in this paper can make the system better performance than the method proposed in [13].

If the model is mismatchable, which the gain of object K changes by 50% and the time constant T changes by 20%, the speed response curves and performance parameters are shown in Fig. 8 and table 2. From the foregoing, with the model parameters changing, the system has stronger robustness when the controller uses the setting method proposed in this paper than the method of [13].

6.2 The analysis of experiment results

The experimental platform with the TMS320LF2407 DSP chip as its core is the specialized platform of doing motor experiment. The main modules of induction motor speed control system based on DSP chip are a induction motor, a feedback module, a power module and a digital control module. In this paper, the induction motor is controlled by the designed fractional-order internal model controller driving IGBT module. Setting parameters of controller, the given value and real-time monitoring are through the PC to finish. The experimental platform is also a very good platform to observe the control performance of controller. This experiment platform uses software program to realize the functions of hardware and implements the induction motor current and speed double closed loop control through the software program, which makes the current and speed controller are fully digitization.

In this power, experimental motor is Sieber (three-phase AC induction motor). The motor parameters of experimental are chosen the same as simulating. When speed is given 900 rpm, the experimental images of rotation speed and q, d shaft current are shown in Fig. 9, Fig. 10 and Fig. 11.

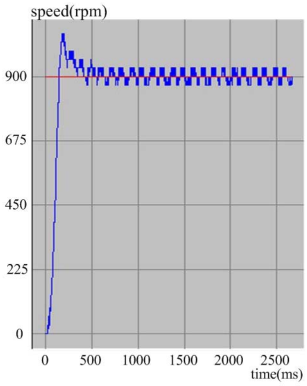

Fig. 9 the speed curve.

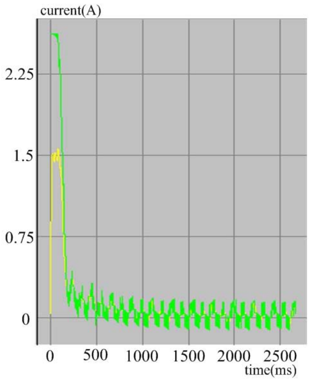

Fig. 10 q-axes current curve.

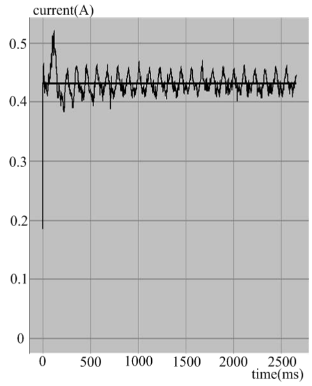

Fig. 11 d-axes current curve.

Fig. 9 is the speed of response curves. Fig. 10 and Fig. 11 are the current response curves of q and d shaft. From the experimental images of speed and current response curves, the controller designed in this paper can make induction motor start quickly and run very smoothly. Although it also can track the given speed, the speed curves have some steady state error. The starting current of q axis is bigger. But it decreases gradually with the increasing of rotational speed, which can effectively track the output of speed controller. The current of d shaft is floating up and down the given value, which can satisfy the requirement of speed regulating system.

7 Conclusion

The parameter setting method of the fractional order controller is introduced, which is based on internal model principle in this paper, and the fractional-order controller is successfully used in induction motor speed control system. Comparing with those conventional fractional order $PI^{\lambda}D^{\mu}$, fractional-order internal model controller have only two adjustable parameters and can use the method of phase margin to set parameters, which makes choosing the parameters of controller not blind. The simulation and experimental results also show that the system will have a good control performance with the proposed method.

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