A Weighted OFDM Signal Scheme for Peak-to-Average Power Ratio Reduction of OFDM Signals

Chang Eon Shin, Kyung Soo Rim, and Youngok Kim

***Abstract*—In this paper, a peak-to-average-ratio (PAPR) reduction scheme based on a weighted orthogonal frequency-division multiplexing (OFDM) signal is proposed to reduce the PAPR without distortion in removing the weight at the receiver side. In the proposed scheme, a weight is imposed on each discrete OFDM signal via a certain kind of a bandlimited signal, and an OFDM signal formed with the weighted discrete data is then considered before a high power amplifier (HPA), whereas the original signal can be recovered completely at the receiver side. Meanwhile, the time duration needed to transmit the weighted OFDM signal is the same as the time duration for the original OFDM signal. The effectiveness of the proposed scheme is evaluated with computer simulations. According to numerical results, the PAPR of the weighted OFDM signal is smaller than that of the clipping and filtering (C&F) method, and the bit-error-rate (BER) performance of the weighted OFDM system is improved compared with the C&F method. Here, the proposed method is simpler than the C&F method.**

***Index Terms*—Convolution, orthogonal frequency-division multiplexing (OFDM), peak-to-average-power ratio (PAPR), weighted data.**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) is a transmission technique that modulates multiple carriers simultaneously. Although their spectra overlap, the transmitted multiple carriers can be demodulated orthogonally, provided that correct time windowing is used at the receiver. Since the OFDM-based system has high spectral efficiency and is robust against intersymbol interference and frequency-selective fading channels, it has been widely chosen for European digital audio/video broadcasting and wireless local/metropolitan area network standards, and now, it is used in most broadband wireless communication systems.

However, one of the major problems of OFDM-based systems is the high peak-to-average-power ratio (PAPR) of a transmitted signal, which causes a distortion of a signal at the nonlinear high-power amplifier (HPA) of a transmitter. Thus, the power efficiency of the HPA is seriously limited to avoid nonlinear distortion; otherwise, the high PAPR results in significant performance degradation. Because of the practical importance of this problem, a number of algorithms for reducing the high PAPR have been developed, such as clipping and filtering (C&F) [1]–[3]; coding [4]–[7]; adaptive symbol selection, such as selected mapping; partial transmit sequence and interleaving [8]–[10]; tone reservation/injection [11], [12]; active signal constellation extension [13], companding [14]–[16]; and others.

In this paper, a PAPR reduction scheme based on a weighted OFDM signal is proposed to reduce the PAPR without distortion in removing the weight at the receiver side. This method is motivated by a circular convolution process, i.e., the modulated OFDM signal is convoluted with a certain kind of signal Φ for smoothing the peak of the OFDM signal before the HPA. Here, we choose the signal Φ to satisfy that the Fourier transform φ of Φ has no zero on the real line. The convoluted signal can be written as a simple weighted OFDM signal. When the discrete data $\{a_k\}_{k=0}^{N-1}$ is given, we consider weighted data $\{a_k\varphi(k)\}_{k=0}^{N-1}$ and form an OFDM signal with this weighted discrete data. Then, this weighted OFDM signal is the same as the given convoluted signal.

Since weight φ is nonuniform, the bit-error-rate (BER) performance could be degraded. In practice, to improve the BER performance, we modify the weight by adding a suitable positive constant to the original weight. The PAPR of the weighted OFDM signal with the modified weight is smaller than that of the C&F method, and the BER performance is improved compared with the C&F method. The effectiveness of the proposed scheme is evaluated with computer simulations.

In this weighted OFDM method with modified weight, the time duration needed to transmit the weighted OFDM signal is the same as the time duration for the original OFDM signal. Moreover, the original discrete data can be recovered completely at the receiver side with additional $2N$ complex multiplications of computational complexity without extra cost in transmission.

The weighted OFDM scheme was introduced in [17], where the Gaussian function, sine function, and some other functions were used as weighted functions. In [17], when the noise is not present, the PAPR of the weighted OFDM system with Gaussian weight is reduced remarkably. As mentioned in the conclusion, however, the noise was not also considered for BER performance. If the additive Gaussian noise is considered, the BER performance of the weighted OFDM system with Gaussian weight will be even degraded. In this paper, we suggest the weighted OFDM system with modified weight to improve the BER performance, and we also provide the condition for a function to be a weight function and a mathematical reason for the merit of the weighted OFDM system derived from a circular convolution system.

The rest of this paper is organized as follows. In Section II, the considered system is briefly described. In Section III, we provide the weighted OFDM signal motivated by the convolution method. In Section IV, the effectiveness of the proposed scheme is evaluated with simulation results. Finally, the conclusions are given in Section V.

II. System Description

Orthogonal multicarrier modulation is an efficient method of data transmission over channels with frequency-selective fading. This method has a relatively simple implementation based on the inverse fast Fourier transform (IFFT).

The simplified block diagrams for an OFDM system with the convolution scheme and the proposed weighted scheme are shown in Fig. 1. As described in Fig. 1(a), the modulated data stream is carried on the multicarriers by the IFFT, and the convolution block reduces the PAPR of signal, which is corresponding to the weight block of the proposed scheme, as shown in Fig. 1(b). In the following block, the cyclic prefix is added before the HPA.

For a discrete data $\{a_k\}_{k=0}^{N-1}$, multicarrier-modulated signal $x_N(t)$ on $[0, NT]$ is represented by

$$x_N(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} a_k e^{j2\pi f_k t} \tag{1}$$

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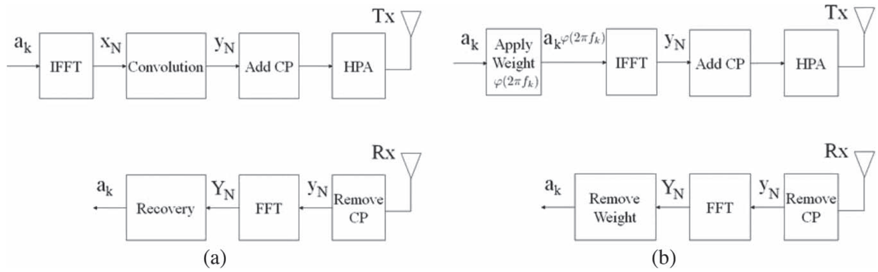

Fig. 1. Simplified block diagrams for an OFDM system with (a) convolution scheme and (b) proposed weighting scheme.

where N is the number of subcarriers, T is the original symbol period, $\Delta f = 1/NT$, and $f_k = k\Delta f$, $k = 0, \ldots, N-1$. The PAPR of x_N over the time interval $[0, NT]$ is defined by

$$\mathrm{PAPR}(x_N) = \frac{\max_{0\le t\le NT} |x_N(t)|^2}{E\left(|x_N(t)|^2\right)} \tag{2}$$

where $E(\cdot)$ denotes the expectation operator.

III. Weighted Orthogonal Frequency-Division Multiplexing System

Here, we provide the weighted OFDM signal, where the weight is derived from a suitable bandlimited signal having no zero on the real line. This method is motivated by a convolution method.

Taking the circular convolution between the multicarrier-modulated signal x_N and a suitable signal Φ having compact support, the PAPR of the convoluted signal can be reduced. In fact, for $p \in \mathbb{R}$ with $1 \le p \le \infty$, from Young's inequality (see [18]), $\|x_N * \Phi\|_p \le \|x_N\|_1 \|\Phi\|_p$, and $x_N * \Phi$ belongs to L^p although $x_N \in L^1$, where $\|f\|_p = (\int_{\mathbb{R}} |f(x)|^p dx)^{1/p}$ and the space $L^p = \{f : \|f\|_p < \infty\}$. For $p > 1$, since L^p is more tempered than L^1, essentially, the PAPR of the convoluted signal can be reduced. Simultaneously, we should consider carefully Φ to sustain the size of the expectation of $x_N * \Phi$.

First, we consider the convolution method and then derive the corresponding weighted OFDM signal.

A. Convolution Method

The Fourier transform $\mathcal{F}[f]$ of f is defined by

$$\mathcal{F}[f](\xi) := \int_{\mathbb{R}} f(x) e^{-j\xi x} dx \tag{3}$$

if the integral exists. The inverse Fourier transform $\mathcal{F}^{-1}[F]$ of F is defined by

$$\mathcal{F}^{-1}[F](x) := \frac{1}{2\pi} \int_{\mathbb{R}} F(\xi) e^{jx\xi} d\xi \tag{4}$$

provided that the integral exists. Then $\mathcal{F}^{-1}[\mathcal{F}[f]] = f$, when f and $\mathcal{F}[f]$ are integrable, and

$$\mathcal{F}\left[\mathcal{F}[f]\right] = 2\pi \check{f} \tag{5}$$

where $\check{f}(x) = f(-x)$.

We consider signal φ as

$$\varphi(x) = \frac{1 - \mathrm{sinc}(x)}{\pi^2 x^2} \tag{6}$$

where

$$\mathrm{sinc}\, x = \begin{cases} \frac{\sin \pi x}{\pi x}, & x \ne 0 \\ 1, & x = 0 \end{cases}.$$

By direct computation, the Fourier transform $\Phi := \mathcal{F}[\varphi]$ of φ is given by

$$\Phi(\xi) = \begin{cases} \frac{1}{2}\left(1 - \frac{|\xi|}{\pi}\right)^2, & |\xi| \le \pi \\ 0, & \text{otherwise.} \end{cases}$$

The signal φ is a bandlimited signal with bandwidth π, has no zero on the real line, and

$$\check{\varphi} = \varphi. \tag{7}$$

For more information about φ, see the Appendix at the end of this paper.

Consider the circular convoluted signal as follows:

$$y_N(t) := \frac{1}{2\pi} x_N * \Phi(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} x_N(t-\xi)\Phi(\xi) d\xi. \tag{8}$$

Taking the Fourier transform in (8), we have by (5) and (7) that

$$\mathcal{F}[y_N] = \frac{1}{2\pi} \mathcal{F}[x_N]\mathcal{F}[\Phi] = \mathcal{F}[x_N]\varphi \tag{9}$$

where $\mathcal{F}[x_N]$ and $\mathcal{F}[y_N]$ are the Fourier transforms in the sense of distribution. Since φ has no zero on the real line and $\mathcal{F}[x_N](\xi) = (2\pi/\sqrt{N}) \sum_{k=0}^{N-1} a_k \delta(\xi - 2\pi f_k)$, we can recover the discrete data so that. for $k = 0, \ldots, N-1$, we have

$$a_k = \frac{\sqrt{N}\mathcal{F}[x_N](2\pi f_k)}{2\pi} = \frac{\sqrt{N}\mathcal{F}[y_N](2\pi f_k)}{2\pi \varphi(2\pi f_k)}. \tag{10}$$

B. Weighted OFDM System

We show that the convoluted signal in (8) can be written as a simple weighted OFDM signal y_N.

Observing by (5), (7) and (8) that

$$\int_{-\pi}^{\pi} e^{j2\pi f_k (t-\xi)} \Phi(\xi) d\xi = 2\pi \varphi(2\pi f_k) e^{j2\pi f_k t}$$

the convoluted signal in (8) can be expressed as the following weighted OFDM signal:

$$y_N(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} a_k \varphi(2\pi f_k) e^{j2\pi f_k t}, \qquad 0 \le t \le NT. \tag{11}$$

C. Weighted OFDM System With Modified Weight

The demerit of the weighted OFDM signal in (11) is the degradation of BER performance since the weight φ is nonuniform. To overcome

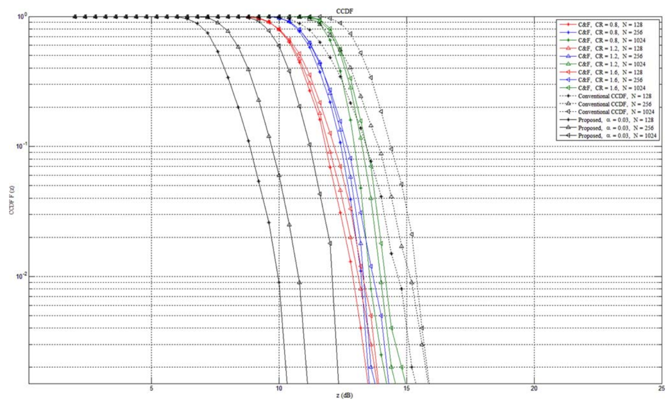

Fig. 2. CCDF of the C&F and proposed methods for $N = 128, 256, 1024$.

this obstacle, we consider the modified weight with a positive constant α as follows:

$$\varphi_\alpha(x) = \varphi(x) + \alpha/\log N \tag{12}$$

where α is a shift parameter, and $\log N$ is obtained by experiment. Then, $\varphi = \varphi_0$. In the weighted OFDM signal in (11), we replace weight φ with φ_α for a suitable positive constant α to get the weighted OFDM signal, i.e.,

$$z_N(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} a_k \varphi_\alpha(2\pi f_k) e^{j2\pi f_k t}, \qquad 0 \le t \le NT \tag{13}$$

as a transmitted signal instead of x_N in (1).

In system (13), weight $\varphi_\alpha(2\pi f_k)$ is imposed on the discrete data a_k, $k = 0, \ldots, N-1$, and we form an OFDM signal with the weighted discrete data $\{a_k \varphi_\alpha(2\pi f_k)\}_{k=0}^{N-1}$ to get weighted OFDM signal z_N. We transmit weighted OFDM signal z_N for the same time duration $[0, NT]$ as the original OFDM signal.

We note that weight φ is positive on the real line; therefore, the modified weight φ_α is positive on the real line. Since $\varphi_\alpha(2\pi f_k) \neq 0$ for any $k = 0, \ldots, N-1$, the discrete data $\{a_k\}_{k=0}^{N-1}$ can be completely recovered.

The PAPR of the weighted OFDM signal z_N is given by

$$\text{PAPR}(z_N) = \frac{\max_{0 \le t \le NT} |z_N(t)|^2}{E\left(|z_N(t)|^2\right)}. \tag{14}$$

In the following, we provide the simulation results showing that the PAPR of the weighted OFDM signal with modified weight φ_α is smaller than that of the C&F method (see Fig. 2), and the BER performance of the weighted OFDM system with modified weight φ_α is improved compared with the C&F method (Fig. 3). We note that as α increases, due to the modification of weight, the BER performance is improved, whereas the complementary cumulative distribution function (CCDF) grows slightly.

In (13), we can recover discrete data $\{a_k \varphi_\alpha(2\pi f_k)\}_{k=0}^{N-1}$ by the conventional method of the OFDM system. Since by dividing the given discrete data by $\varphi_\alpha(2\pi f_k)$ we can obtain the original discrete data, the weighted OFDM system is not expected to cause any computational complexity in recovering the original discrete data. In fact, $2N$ complex multiplications are additionally needed compared with the original OFDM method.

We note that a sufficient condition for a signal $\tilde{\varphi}$ to be a proper weight is that $\tilde{\varphi}(2\pi f_k) \neq 0$ for any $k = 0, \ldots, N-1$. We expect that the performance of the weighted OFDM system corresponding to $\tilde{\varphi}$ depends on the smoothness of the Fourier transform of $\tilde{\varphi}$.

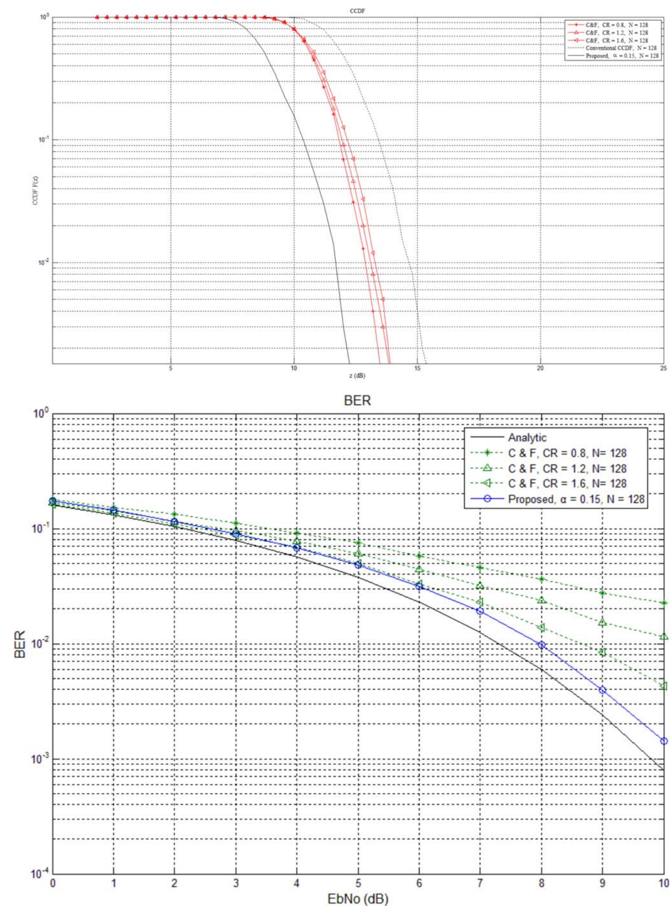

Fig. 3. CCDF and BER of the C&F and proposed methods for $N = 128$.

IV. Simulation Results

The performance of this proposed scheme is analyzed through the simulations. In the simulations, 10^3 quadratic-phase-shift-keying (QPSK)-modulated OFDM symbols were randomly generated.

Fig. 2 shows the CCDFs of the C&F method and the proposed method for $N = 128, 256, 1024$. The proposed method is simulated with a fixed shift parameter $\alpha = 0.03$, and several C&Fs are simulated with various clipping ratios $\text{CR} = 0.8, 1.2, 1.6$, respectively. As shown in the figure, the proposed scheme can reduce the PAPR around 3 dB for $N = 128$ and 2 dB for $N = 1024$, respectively, at the 1% of the CCDF, compared with the C&F scheme. Note that the PAPR of the original OFDM signal exceeds 14.8 dB for $N = 128$ and 16 dB for $N = 1024$, respectively. In Figs. 2 and 3, since the results induced by quadratic-amplitude modulation mapping are almost the same as those induced by QPSK mapping, here, we provide only the results induced by QPSK mapping.

Fig. 3 compares the C&F method with the proposed method for CCDFs and BER performance over the additive White Gaussian noise channel together. As shown in the figure, the BER performance and the CCDF of the proposed method with $\alpha = 0.15$ are superior to those of the C&F method for $\text{CR} = 0.8, 1.2, 1.6$ when $N = 128$ is fixed.

V. Conclusion

A PAPR reduction scheme based on a weighted OFDM signal has been proposed to reduce the PAPR without data distortion in removing the weight at the receiver side in the mathematical view. To reduce the peak of the OFDM signal, a bandlimited signal φ, which is not zero on the set $\{2\pi f_k\}_{k=0}^{N-1}$, is introduced, and we form weight

$\varphi_\alpha = \varphi + \alpha / \log N$ for a suitable positive constant α. We consider a weighted discrete data to form a weighted OFDM signal, which is defined on the same time interval as the original OFDM signal, before the HPA, where the weights are imposed by using signal φ_α. It is shown that the PAPR of this weighted OFDM method is smaller than that of the C&F method, and the BER performance is improved compared with the C&F method.

APPENDIX

We introduce some properties of the signal φ in (4).

The signal φ in (4) was found in the process of sampling expansion for a bandlimited signal, which has a polynomial growth on the real line.

Note that signal φ is a bandlimited signal having a polynomial decay on the real line, i.e., there exists $C > 0$, such that, for any real number x, $\varphi(x) \leq (C/(1+|x|)^2$.

For example, if f is a bandlimited signal and there exists $C' > 0$, such that, for any real number x, $|f(x)| \leq C'(1+|x|)$,, then the multiplication $f \cdot \varphi$ is a bandlimited signal with finite energy; therefore, we can apply the Shannon sampling theorem to $f \cdot \varphi$, and we can obtain the sampling expansion of the signal f on the real line.

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Adaptive-Rate Transmission With Opportunistic Scheduling in Wireless Networks

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***Abstract*—In this paper, we propose an adaptive-rate transmission scheme with opportunistic scheduling in a two-hop relay network over a Rayleigh fading environment. We assume that only channel state information (CSI) at receivers is available in a decentralized network, where n source-to-destination pairs are completely independent and m half-duplex relays cooperate by exchanging their selected source IDs at the beginning of the two-hop transmission. In the limit of large n and fixed m, the system throughput of the scheme scales as $(m/2)$ log log n. It is shown that this is the same achievable scaling even with perfect CSI assumptions at transmitters and full cooperation among nodes. Furthermore, it is shown that the optimal scaling of m is $\Theta(\log n)$, under which a linear increase in throughput with m is obtained. A closed-form delay expression of the proposed scheme is also presented.**

***Index Terms*—Cooperative relaying, opportunistic scheduling, two-hop communication.**

I. INTRODUCTION

The demand for more efficient and reliable communication networks has given rise to a number of innovative systems and network architectures, such as *ad hoc* networks, cognitive radio (CR), relay extensions for cellular networks, and wireless sensor networks. In their seminal work [1], Gupta and Kumar focused on the study of system throughput. Numerous schemes have since been proposed that apply different assumptions to the channel state information (CSI) and levels of cooperation among communicating nodes [2]–[5]. These studies have contributed to the understanding of system throughput in wireless networks. One common thread among these studies is a focus on the scaling of the system throughput with fixed-rate transmissions. For some network architectures, such as *ad hoc*, sensor, and CR networks, using adaptive-rate transmission may provide better system performance, due to its ability to change transmission parameters. In particular, the transmit power and modulation level can be adjusted

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