PAPR Reduction Techniques with Hybrid SLM-PTS Schemes for OFDM Systems

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***Abstract*—In general, the high peak-to-average power ratio (PAPR) of transmitted signals for OFDM systems reduces the system efficiency and hence increases the cost of the radio frequency (RF) power amplifier. In this paper, a modified hybrid algorithm is developed to obtain better PAPR reduction performance and reduce computational complexity compared with the conventional hybrid scheme. This proposed algorithm combines selected mapping (SLM) with partial transmit sequence (PTS) strategies, and further employs linear addition and exchange of various PTS sub-blocks to create more alternative OFDM signal sequences. As a result, with the same numbers of IFFT and phase rotation sequences, our proposed algorithm has the potentials to provide better PAPR reduction performance with lower computational complexity.**

***Index Terms*—OFDM, PAPR, SLM, PTS.**

I. INTRODUCTION

In wireless communication systems, the orthogonal frequency division multiplexing (OFDM) [1]–[4] technique is a widely popular and attractive scheme for high-data-rate transmission because it can cope with frequency-selective fading channel. The modulators and demodulators of OFDM systems can be simply implemented by employing inverse fast Fourier transform (IFFT) and FFT to make the overall system efficient and effective. Nowadays, it has been adopted as a powerfully potential candidate for next-generation mobile communications systems.

For OFDM-based systems, one of the main disadvantages is high PAPR problem. This phenomenon results from that in the time domain, an OFDM signal is the superposition of many narrowband subcarriers. At certain time instances, the peak amplitude of the signal is large and at the other times is small, that is, the peak power of the signal is substantially larger than the average power of the signal. The influence of high PAPR reduces system efficiency and then increases the cost of the RF power amplifier. Therefore, how to find a solution to reduce high PAPR effectively is one of the most important implementation issues in OFDM communications.

As a review of previous literature, the multiple signal representation is one of well-known PAPR reduction techniques for OFDM systems. It has been described in [5]–[13] particularly. Several helpful schemes related to SLM-based and PTS-based techniques have been proposed for improving PAPR reduction performance or reducing the computational complexity. Those techniques included the conventional hybrid method [12] and the modified SLM scheme [13]. Based on the preceding survey results, a novel modified hybrid algorithm combining the additional hybrid with switching hybrid schemes is proposed to reduce the number of IFFT and obtain a significant PAPR reduction performance in OFDM systems.

The remainder of this paper is organized as follows. In Section II, the PAPR issue of OFDM systems is briefly described. The conventional hybrid scheme for PAPR reduction is reviewed and introduced in Section III. The proposed modified hybrid algorithm combining the additional hybrid with switching hybrid schemes is developed and analyzed in Section IV. Finally, some comparative simulation results are given in Section V, and the concluding remarks are drawn in Section VI.

II. OFDM PAPR DESCRIPTIONS

A discrete-time OFDM model with N subcarriers is considered. With the linear property of the N narrowband subcarriers, the discrete-time OFDM signals can be written as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi kn}{N}} \tag{1}$$

where $n = 0, 1, \ldots, N-1$. For simplicity, we can have $\mathbf{x}$ = IFFT $\{\mathbf{X}\}$, where $\mathbf{X} = [X(0), ..., X(N-1)]^T$ and $\mathbf{x} = [x(0), ..., x(N-1)]^T$. Because of the statistical independency of the transmitted subcarriers, the time domain OFDM samples $x(n)$ are approximate Gaussian distribution. The definition of PAPR can be written as

$$\text{PAPR} = \frac{\max\limits_{0\le n\le N-1} |x(n)|^2}{E\{|x(n)|^2\}} \tag{2}$$

where $E\{\cdot\}$ denotes expectation operator.

In the literature, the complementary cumulative distribution function (CCDF) of the PAPR is one of the most frequently used performance measures for PAPR reduction techniques. The CCDF of PAPR denotes the probability that the PAPR of an OFDM symbol exceeds a given threshold PAPR$_o$, that is, the CCDF of PAPR can be written as

$$\text{CCDF}(\text{PAPR}_o) = Pr(\text{PAPR} > \text{PAPR}_o) \tag{3}$$

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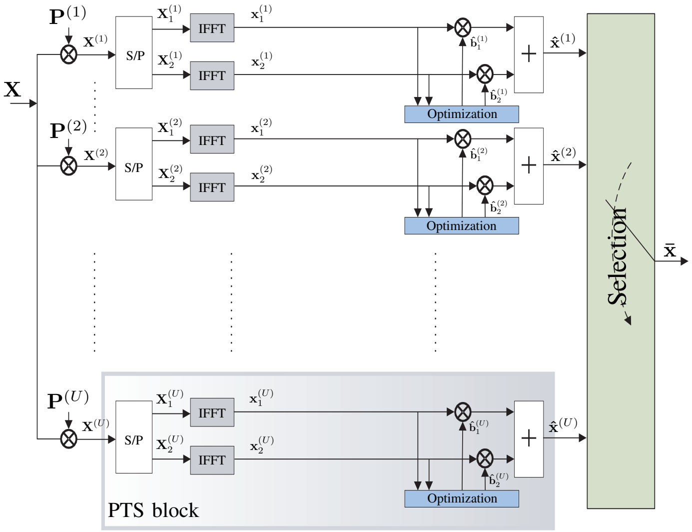

Fig. 1. The block diagram of conventional hybrid scheme.

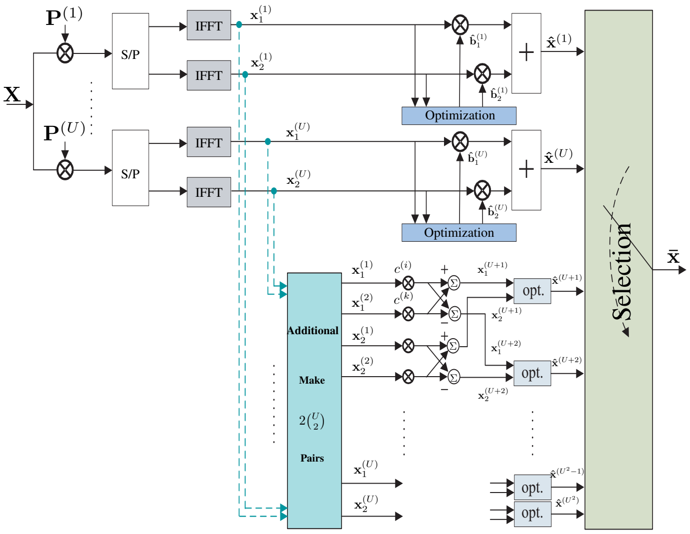

Fig. 2. The block diagram of additional hybrid scheme.

In general, for OFDM systems with Gaussian time domain samples, the CCDF of PAPR can be written as

$$Pr(\text{PAPR} > \text{PAPR}_o) = 1 - (1 - e^{-\text{PAPR}_o})^N \tag{4}$$

III. CONVENTIONAL HYBRID SCHEME

In the beginning, the conventional hybrid (CH) method combining the SLM with PTS schemes is investigated. The strategy was first explicitly proposed by P. A Pushkarev in [12]. The block diagram of the CH method is shown in Fig. 1. The original OFDM symbol is multiplied with the U phase rotation sequences, and then each of the new OFDM symbols is partitioned into V pairwise disjoint sub-blocks. Those OFDM sub-block values are calculated by each optimization of PTS blocks. For simplicity and without loss of generality, $V = 2$ is always considered in this paper. Each signal $\hat{\mathbf{x}}^{(u)}$, where $u = 1, \ldots, U$, with the lowest PAPR is selected by each optimization block. They can be written as

$$\{\hat{\mathbf{b}}_1^{(u)}, \hat{\mathbf{b}}_2^{(u)}\} = \underset{\{\mathbf{b}_1^{(u)}, \mathbf{b}_2^{(u)}\}}{\text{argmin}} \left\{ \sum_{v=1}^{2} \mathbf{b}_v^{(u)} \mathbf{x}_v^{(u)} \right\} \tag{5}$$

$$\hat{\mathbf{x}}^{(u)} = \sum_{v=1}^{2} \hat{\mathbf{b}}_v^{(u)} \mathbf{x}_v^{(u)} \tag{6}$$

where $1 \le u \le U$.

By the selection block, the relatively lower PAPR can be obtained from those lowest PAPR values of each PTS block. Because those lowest PAPR values of each PTS block are statistically independent to each other, the CCDF of CH scheme can be written as

$$\text{CCDF}_{\text{CH}} = \Big(Pr\Big(\text{PAPR}_{PTS} > \text{PAPR}_o\Big)\Big)^U \tag{7}$$

In order to recover transmitted data information, the receiver must have the knowledge of side information. Because the CH signal must include the side information of SLM and the side information of PTS, the number of required side information bits can be written as

$$N_{\text{CH}} = \log_2 U + (V-1)\log_2 W \tag{8}$$

where W is the number of allowed phase rotation factors. In (8), the first term expresses the SLM required side information bits and the second term is the additional bits from the PTS algorithm.

IV. PROPOSED HYBRID SCHEMES

A. *Additional hybrid scheme*

In order to improve the PAPR reduction performance in CH scheme, we have to generate a large number of alternative OFDM signal sequences without increasing the number of IFFT to avoid high computational complexity. Here, a new additional hybrid (AH) scheme by combining the modified SLM scheme with CH scheme. The system performance is desirable that the number of IFFT is reduced but the PAPR reduction performance is not compromised. The block diagram of AH scheme is shown in Fig. 2.

Clearly, the first U signals $\hat{\mathbf{x}}^{(u)}$, where $u = 1, \ldots, U$, are the same as the signals (6) in the CH scheme. Furthermore, the alternative OFDM signal sequences are generated by the linear combination of the sub-block signals from different PTS blocks after IFFT operation. Using the linear property of Fourier transform, the linear combination of these sequences can be obtained by

$$\mathbf{x}_v^{(u)} = c^{(i)}\mathbf{x}_v^{(i)} + c^{(k)}\mathbf{x}_v^{(k)} \tag{9}$$

where $U + 1 \le u \le U^2$, $1 \le i, k \le U$, $1 \le v \le 2$, and $c^{(i)}$ and $c^{(k)}$ are some coefficients to be chosen later. That is to say, if we have OFDM signal sequences $\mathbf{x}_v^{(i)}$ and $\mathbf{x}_v^{(k)}$, the other alternative OFDM signal sequences in (9) can be obtained without performing IFFT operation. Now, we would investigate how to make each element of $\mathbf{x}_v^{(i)}$ and $\mathbf{x}_v^{(k)}$ to have

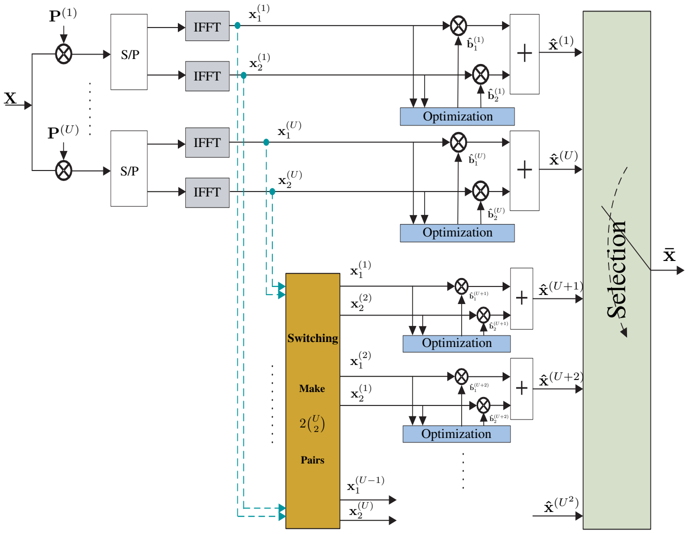

Fig. 3. The block diagram of switching hybrid scheme.

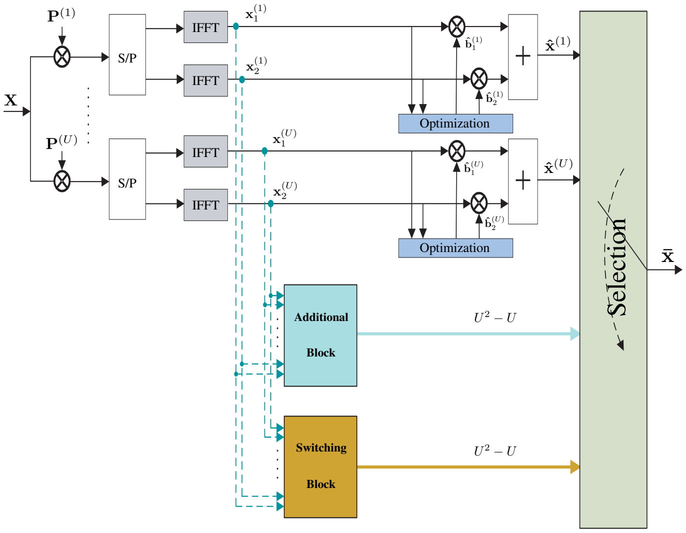

Fig. 4. The block diagram of modified hybrid scheme.

unit magnitude under the condition that each element of the phase sequences $\mathbf{P}^{(i)}$ and $\mathbf{P}^{(k)}$ has unit magnitude. Basically, the elements of the sequence $\mathbf{x}_v^{(i)}$ and $\mathbf{x}_v^{(k)}$ have unit magnitude if the following conditions are satisfied:

- $c^{(i)} = \pm(1/\sqrt{2})$ and $c^{(k)} = \pm(1/\sqrt{2})j$, and
- Each element of $\mathbf{P}^{(i)}$ and $\mathbf{P}^{(k)}$ takes the value in ±1.

Since $|c^{(i)}|^2 = |c^{(k)}|^2 = 1/2$, the average power of $\mathbf{x}_v^{(u)}$ is equal to one half of the sum of average power of $\mathbf{x}_v^{(i)}$ and $\mathbf{x}_v^{(k)}$. From U binary phase rotation sequences, we can obtain $2\binom{U}{2}$ excessive pair sub-blocks sequences, thus, there are total U^2 pair sub-blocks sequences for AH scheme. Then, the alternative OFDM signal of lowest PAPR in AH scheme can be written as

$$\{\hat{\mathbf{b}}_1^{(u)}, \hat{\mathbf{b}}_2^{(u)}\} = \underset{\{\mathbf{b}_1^{(u)}, \mathbf{b}_2^{(u)}\}}{\text{argmin}} \left\{ \mathbf{b}_1^{(u)}\mathbf{x}_1^{(u)} + \mathbf{b}_2^{(u)}\mathbf{x}_2^{(u)} \right\} \tag{10}$$

$$\hat{\mathbf{x}}^{(u)} = \hat{\mathbf{b}}_1^{(u)}\mathbf{x}_1^{(u)} + \hat{\mathbf{b}}_2^{(u)}\mathbf{x}_2^{(u)} \tag{11}$$

where $U + 1 \leq u \leq U^2$.

We have to select and transmit the resulting OFDM signal sequence $\bar{\mathbf{x}}$, which has the minimum PAPR among the whole OFDM signal sequences of overall lowest PAPR $\hat{\mathbf{x}}^{(u)}$ sequences, which are composed by $\{\mathbf{x}_1^{(u)}, ..., \mathbf{x}_V^{(u)}\}$ after each optimization operation. The number of required side information bits for transmitter can be written as

$$N_{\text{AH}} = \log_2 U^2 + (V-1)\log_2 W \tag{12}$$

B. Switching hybrid scheme

Instead of generating alternative OFDM sequences with linear combination, a new switching hybrid (SH) scheme by combining the switching technique with the CH scheme. The system performance is desirable that the number of IFFT is reduced but the PAPR reduction performance is not compromised. The block diagram of SH scheme is shown in Fig. 3.

By the switching block, we can use original U pairs $\{\mathbf{x}_1^{(u)}, \mathbf{x}_2^{(u)}\}$ to generate excessive $2\binom{U}{2}$ pairs of OFDM sequences without increasing the number of IFFT units. Thus, there are total U^2 pairs $\{\mathbf{x}_1^{(u)}, \mathbf{x}_2^{(u)}, ..., \mathbf{x}_1^{(u^2)}, \mathbf{x}_2^{(u^2)}\}$ are operated by each optimization unit. Obviously, the first U signals $\hat{\mathbf{x}}^{(u)}$, where $u = 1, \ldots, U$, are the same as the signals (6) in the CH scheme. After the optimization blocks, the other alternative OFDM sequences with lowest PAPR $\hat{\mathbf{x}}^{(u)}$ can be written as

$$\{\hat{\mathbf{b}}_1^{(u)}, \hat{\mathbf{b}}_2^{(u)}\} = \underset{\{\mathbf{b}_1^{(u)}, \mathbf{b}_2^{(u)}\}}{\text{argmin}} \left\{ \mathbf{b}_1^{(u)}\mathbf{x}_1^{(i)} + \mathbf{b}_2^{(u)}\mathbf{x}_2^{(k)} \right\} \tag{13}$$

$$\hat{\mathbf{x}}^{(u)} = \hat{\mathbf{b}}_1^{(u)}\mathbf{x}_1^{(i)} + \hat{\mathbf{b}}_2^{(u)}\mathbf{x}_2^{(k)} \tag{14}$$

where $U + 1 \leq u \leq U^2$, $1 \leq i, k \leq U$ and $i \neq k$. In (14), $\mathbf{x}_v^{(i)}$ and $\mathbf{x}_v^{(k)}$, $i \neq k$ come from different PTS blocks, which are generated by different phase rotation sequences, so that $\mathbf{P}^{(i)}$ and $\mathbf{P}^{(k)}$, where $1 \leq i, k \leq U$, $i \neq k$, can obtain differently alternative OFDM sequences with the minimum PAPR. Noteworthy, the number of required side information bits can be written as

$$N_{\text{SH}} = \log_2 U^2 + (V-1)\log_2 W \tag{15}$$

C. Modified hybrid scheme

In order to further improve the PAPR reduction performance without increasing the number of IFFT, the modified hybrid (MH) algorithm is proposed by combining AH and SH schemes to generate more and more alternative OFDM sequences. Those $\{\mathbf{x}_1^{(u)}, \mathbf{x}_2^{(u)}\}$ pairs, where $1 \leq u \leq U$, are the signal inputs of the additional block and switching block respectively and simultaneously. The block diagram of MH scheme is shown in Fig. 4.

TABLE I

SIMULATION PARAMETERS

Simulation Parameters	**Specifications**
Number of OFDM symbols	1000000
Number of subcarriers	64
Number of sub-blocks	$V = 2$
Phase rotation factors	$\mathbf{b}^{(v)} \in \{\pm 1, \pm j\}$
Phase rotation sequences	$\mathbf{P}^{(u)} \in \{\pm 1\}$
Modulation scheme	QPSK

TABLE II

SIDE INFORMATION AND PERFORMANCE COMPARISON FOR VARIOUS PAPR REDUCTION SCHEMES

Schemes	**PAPR**$_o$ (10^{-3})	**Side information bits**
CH	6.4dB	$\log_2 U + (V-1)\log_2 W$
AH	5.6dB	$\log_2 U^2 + (V-1)\log_2 W$
SH	5.6dB	$\log_2 U^2 + (V-1)\log_2 W$
MH	5.3dB	$\log_2 (2U^2 - U) + (V-1)\log_2 W$

Using the linear property of Fourier transform, the linear combination of U phase rotation sequences can obtain excessive $2\binom{U}{2}$ alternative OFDM sequences. After optimization blocks, those overall lowest PAPR $\hat{\mathbf{x}}^{(u)}$ can be written as the same as (11). Using the switching technique among PTS blocks, the signals of U phase rotation sequences can obtain excessive $2\binom{U}{2}$ alternative OFDM sequences. After optimization blocks, those overall lowest PAPR $\hat{\mathbf{x}}^{(u)}$ can be written as the same as (14).

In the MH scheme, if $V = 2$ and U phase rotation sequences are considered, the original signals $\mathbf{x}_v^{(u)}$ can generate excessive $2\binom{U}{2}$ pairs of sequences respectively and simultaneously by either additional block or switching block. Therefore, there are total $2U^2 - U$ OFDM sequences with the lowest PAPR in the MH scheme. In order to recover the transmitted data information, the number of required side information bits can be obtained by

$$N_{\mathrm{MH}} = \log_2(2U^2 - U) + (V-1)\log_2 W \qquad (16)$$

V. COMPARATIVE SIMULATION RESULTS

The system parameters for comparative simulations are listed in Table I. The PAPR reduction performance with the CH scheme for various values of U is shown in Fig. 5. It shows that the PAPR reduction performance becomes better as the number of U increases. Obviously, in Fig. 6 and Fig. 7, the performance of AH scheme is similar to that of SH scheme. The AH and SH schemes with $U = 2$ and $U = 4$ have almost the same performance compared with the CH scheme with $U = 4$ and $U = 16$, respectively. In Fig. 8, the MH method with $U = 2$ and $U = 4$ has almost the same performance compared with the CH scheme with $U = 6$ and $U = 28$, respectively.

TABLE III

COMPARISON FOR NUMBER OF IFFT WITH SIMILAR PAPR REDUCTION PERFORMANCE

PAPR$_o$ (CCDF$= 10^{-3}$)	**6.1dB**	**5.3dB**
No. of IFFT for CH	12	56
No. of IFFT for MH	4	8

From the simulation results, we can expect that those excessive alternative OFDM signals generated from the original sequences of CH scheme in the AH, SH and MH methods should be almost different from the OFDM sequences in the CH scheme. Therefore, the MH scheme possesses the best PAPR reduction performance with minor increase of computational complexity. The system complexity of various PAPR reduction schemes for OFDM systems is given in Table II, where the number of IFFT is fixed to be 8, that is, $U = 4$, $V = 2$ and $W = 4$.

In Table III, the system complexity with similar PAPR reduction performance is discussed. If CCDF $= 10^{-3}$ is considered, the MH scheme needs only one-third of the number of IFFT compared with the CH scheme when the threshold PAPR$_o$ is 6.1dB. When the threshold PAPR$_o$ is 5.3dB, the MH scheme needs only one-seventh of the number of IFFT. Therefore, the required number of IFFT in the MH scheme is less than that in the CH scheme.

VI. CONCLUDING REMARKS

In general, the PAPR reduction performance becomes better as the number of U increases in CH scheme, but the CH scheme has high computational complexity because of the increase of the number of IFFT. Therefore, based on original signals of CH scheme, several powerful algorithms have been proposed to improve high PAPR reduction performance without increasing the number of IFFT, including AH, SH and MH schemes. The MH scheme can obtain the best PAPR reduction performance by combining the AH with SH schemes. After a number of comparative simulations, the MH scheme has shown that the excellent PAPR reduction performance can be achieved without increasing the number of IFFT.

To sum up, the proposed MH scheme has obtained a superior PAPR reduction performance for OFDM systems. The technique has a better PAPR reduction performance by increasing the number of alternative OFDM sequences. In particular, when the number of IFFT is the same, the MH scheme has the best PAPR reduction compared with CH, AH and SH schemes. Therefore, for the MH scheme, it can expend less IFFT units to obtain similar PAPR reduction performance without the dramatic increase of side information bits.

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REFERENCES

[1] R. van Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*, Boston: Artech House, 2000.

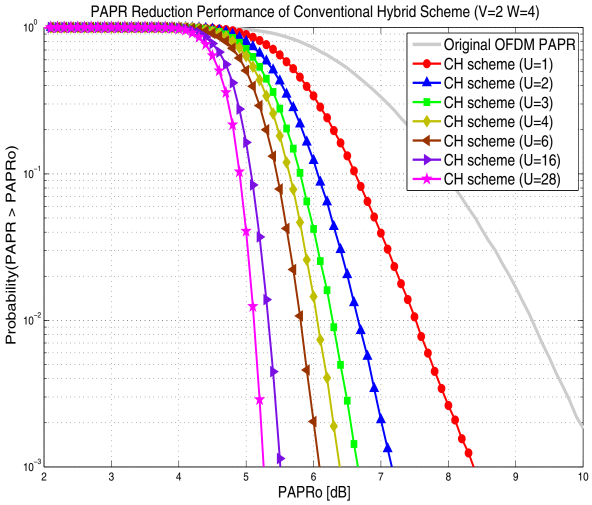

Fig. 5. The PAPR reduction performance of conventional hybrid scheme for OFDM systems.

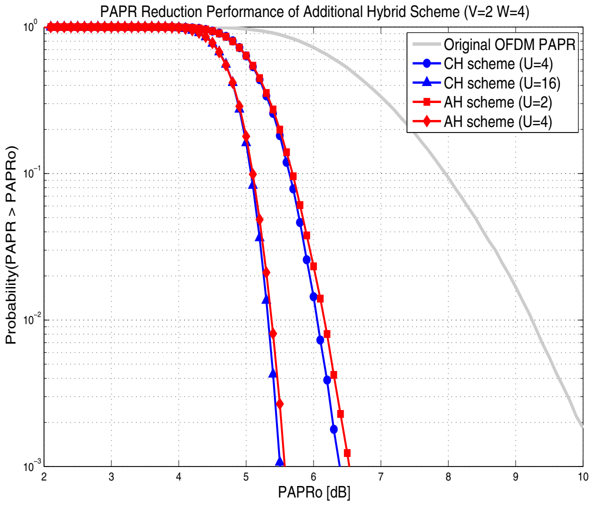

Fig. 6. The PAPR reduction performance of additional hybrid scheme for OFDM systems.

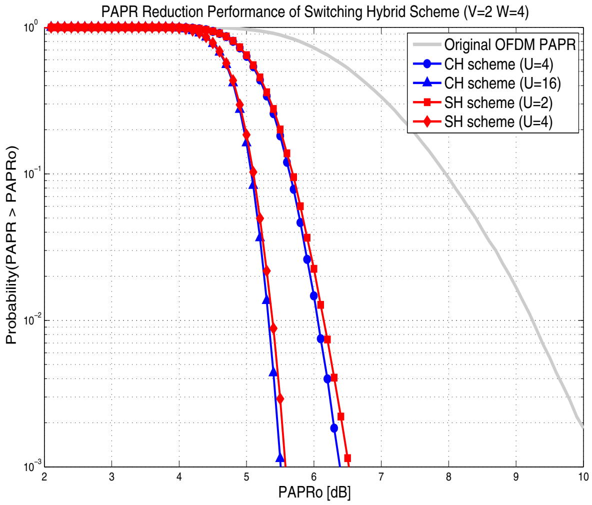

Fig. 7. The PAPR reduction performance of switching hybrid scheme for OFDM systems.

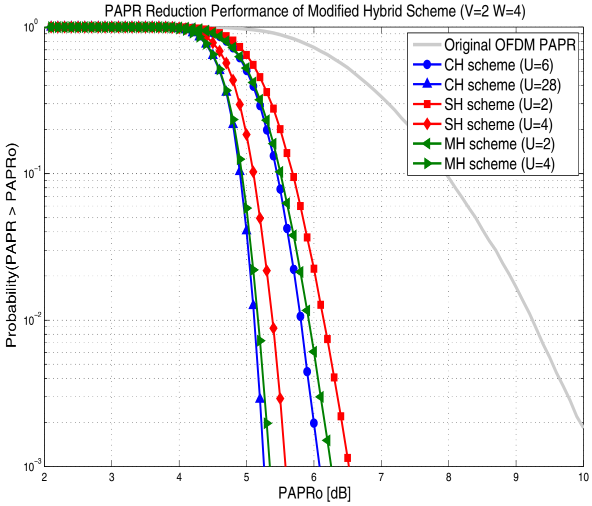

Fig. 8. The PAPR reduction performance of modified hybrid scheme for OFDM systems.

[2] W. Y. Zou and Y. Wu, "COFDM: An Overview," *IEEE Transactions on Broadcasting*, Vol. 41, No. 1, pp. 1-8, Mar. 1995.

[3] R. C. T. Lee, M.-C. Chiu and J.-S. Lin, *Communications Engineering: Essentials for Computer Scientists and Electrical Engineers*, Singapore: Wiley-IEEE Press, 2007.

[4] T. Hwang, C. Yang, G. Wu, S. Li and G. Y. Li, "OFDM and Its Wireless Applications: A Survey," *IEEE Transactions on Vehicular Technology*, Vol. 58, No. 4, pp. 1673-1694, May 2009.

[5] S.-H. Han and J.-H. Lee, "An Overview of Peak-to-Average Power Ratio Reduction Techniques for Multicarrier Transmission," *IEEE Transactions on Wireless Communications*, Vol. 12, No. 2, pp. 56-65, Apr. 2005.

[6] T. Jiang and Y. Wu, "An Overview: Peak-to-Average Power Ratio Reduction Techniques for OFDM Signals," *IEEE Transactions on Broadcasting*, Vol. 54, No. 2, pp. 257-268, Jun. 2008.

[7] R. Bäuml, R. F. H. Fischer and J. B. Huber, "Reducing the Peak-to-Average Power Ratio of Multicarrier Modulation by Selected Mapping," *IEE Electronics Letters*, Vol. 32, No. 22, pp. 2056-2057, Oct. 1996.

[8] S. H. Müller, R. Bäuml, R. F. H. Fischer and J. B. Huber, "OFDM with Reduced Peak-to-Average Power Ratio by Multiple Signal Representation," *Annuals of Telecommunications*, Vol. 52, No. 1-2, pp. 58-67, Feb. 1997.

[9] E. Alsusa and L. Yang, "A Low-Complexity Time-Domain Linear Symbol Combining Technique for PAPR Reduction in OFDM Systems," *IEEE Transactions on Signal Processing*, Vol. 56, No. 10, Oct. 2008.

[10] S. H. Müller and J. B. Huber, "A Novel Peak Power Reduction Scheme for OFDM," *Proceedings of International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'97)*, Vol. 3, Helsinki, Finland, pp. 1090-1094, Sep. 1997.

[11] S. H. Müller and J. B. Huber, "OFDM with Reduced Peak-to-Average Power Ratio by Optimum Combination of Partial Transmit Sequences," *IEE Electronics Letters*, Vol. 33, No. 5, pp. 368-369, Feb. 1997.

[12] P. A. Pushkarev, K.-W. Ryu, K.-Y. Yoo and Y.-W. Park, "A Study on the PAR Reduction by Hybrid Algorithm Based on the PTS and SLM Techniques," *Proceedings of the 57th IEEE Vehicular Technology Conference*, Vol. 2, pp. 1263-1267, 2003.

[13] S.-J Heo, H.-S. Noh, J.-S. No and D.-J. Shin, "A Modified SLM Scheme with Low Complexity for PAPR Reduction of OFDM Systems," *IEEE Transactions on Broadcasting*, Vol. 53, No. 4, pp. 804-808, Dec. 2007.