Performance Analysis of DHT-Based Optical OFDM Using Large-Size Constellations in AWGN

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***Abstract*—We demonstrate that optical OFDM (O-OFDM) based on discrete Hartley transform (DHT) can be used with large-size constellations, furnishing alternative simplified schemes suitable for intensity-modulated direct-detection (IM/DD) systems. Both DC-biased and power-efficient asymmetrically clipped (AC) solutions are analyzed for real constellations from BPSK (binary-phase-shift keying) to 32 PAM (pulse-amplitude modulation). The performance is compared to standard O-OFDM based on fast Fourier transform (FFT) using constellations from 4 QAM (quadrature-amplitude modulation) to 1024 QAM, showing perfect agreement. The analysis of clipping effect evidences the suitability of the proposed AC scheme for adaptive systems.**

***Index Terms*—Optical OFDM, intensity modulated direct detection, asymmetrically clipped optical OFDM, discrete Hartley transform.**

I. Introduction

Multi-carrier transmission techniques, combined with high order modulation formats, represent a promising solution for low-cost and high-speed intensity-modulated direct-detection (IM/DD) systems robust against dispersion [1], [2]. The flexibility and scalability to higher order modulation offered by optical orthogonal frequency division multiplexing (O-OFDM) is unique compared to single carrier techniques, that require complex and costly coherent schemes [3]. A cost-effective implementation of the O-OFDM is the discrete multi-tone modulation (DMT), where the OFDM signal is real-valued [1]. In-phase and quadrature modulation onto an RF carrier is not required, thus the complexity of the electronic design is reduced. Furthermore, direct laser modulation is possible, no laser is needed at the receiver and DD can be simply implemented with commercial components [1], [2]. Such systems can be used for a wide range of applications, e.g. high-speed optical LANs, 10Gb/s Ethernet using multimode fiber, interconnects in data centers and high performance computing [1]-[4]. Additionally, adaptive modulation technique can be applied for performance optimization [2].

To generate a real OFDM signal, the input sequence mapped into a complex constellation is forced to have Hermitian symmetry (HS) [1]. Half of the inverse fast Fourier transform (IFFT) points are used to process the data symbols, while half of the carriers are required to process the complex conjugate vector. An alternative O-OFDM technique, which deals with real signals, is based on the discrete Hartley transform (DHT). In this case, the HS is not required, thus all the transform points carry data symbols and the same data rate can be transmitted using real lower-size constellations [5].

The OFDM signal to be transmitted on an IM system must be unipolar, i.e. real and positive. Positive signals can be obtained by adding a DC bias to the real OFDM signals. However, this is an inefficient solution in terms of optical power, as the bias must be at least twice the signal standard deviation. Moreover, in presence of high negative peaks, the noise due to the clipping at zero level degrades the transmission. Alternatively, asymmetrically clipping (AC) is a power-efficient technique [6]. In fact, if only the odd subcarriers are modulated, the OFDM signals can be clipped and correctly recovered without clipping noise. Both DC-biased and AC techniques can be applied to OFDM signals generated by DHT, with the advantage of yielding a simpler system with low computational complexity, as demonstrated in [5]. Indeed, the DHT is a real transform, thus the additional resources for implementing the HS and reversing the kernel sign, required by the FFT, can be saved; additionally, the equalization does not need complex algebra [7]. The digital signal processing (DSP) at the transmitter and receiver uses the same routine, thanks to the DHT self-inverse property, and the DSP speed can be increased by applying minimum arithmetic complexity fast algorithms [5]. Therefore, a real-time implementation using FHT has reduced complexity and cost.

In this letter, we analyze DHT-based O-OFDM systems using higher order modulation formats, in additive white Gaussian noise (AWGN) channels. Moreover, the clipping effect impact in terms of power efficiency and system performance is evaluated. We compare the simulation results with FFT-based O-OFDM, to furnish an alternative simplified implementation for high-capacity, cost-effective IM/DD systems and to design novel adaptively modulated O-OFDM schemes.

II. DHT-Based O-OFDM: System Model

The IM/DD optical OFDM system that we analyze is indicated in Fig. 1. The inverse (forward) fast Hartley transform (FHT) implements the OFDM modulation (demodulation). Since the DHT is a real transform, if a real constellation is used for the input data mapping, the discrete OFDM signal

$$h(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n) \left[\cos(2\pi kn/N) + \sin(2\pi kn/N)\right] \qquad k = 0, 1, 2, \cdots, N-1, \quad (1)$$

is real; $x(n)$ is the n-th element of the N-length vector of constellation symbols at the input of the IFHT and N is the transform order.

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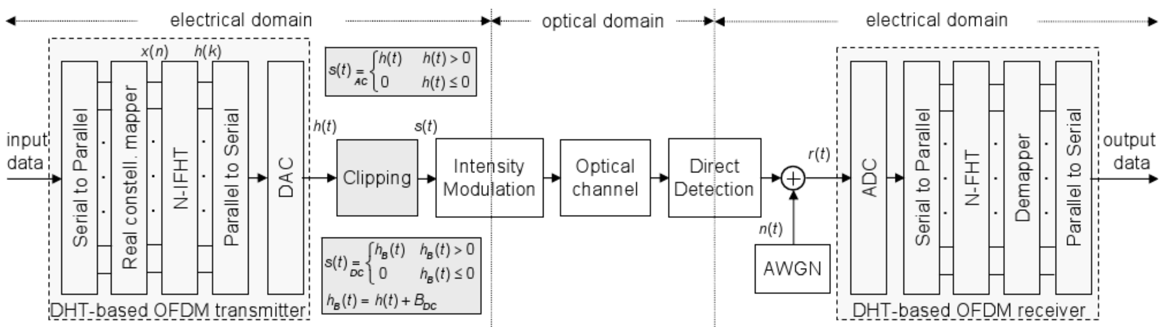

Fig. 1. DHT-based O-OFDM: schematic including Gaussian noise. Clipping is illustrated for AC signals and DC-biased signals with residual negative peaks.

In DMT systems using an FFT of order N, at each parallel processing, the information bit sequence mapped into 4 QAM (quadrature-amplitude modulation) can be transmitted over $N/2$ subcarriers, due to the HS constraint. An O-OFDM system based on an N-order DHT supports the double of information symbols, since all the N transform points are available to transmit independent data. Therefore, to transmit the same information bit sequence per parallel processing, the input bits can be simply mapped into BPSK (binary phase-shift keying) modulation format. Similarly, for larger-size L-QAM constellation, when DHT is used, the size of the real M-PAM (pulse-amplitude modulation) constellation is $M = \sqrt{L}$ [5].

As in DMT systems, only one single digital-to-analog converter (DAC) is required at the transmitter. After DD, the received signal $r(t)$ is converted to digital by one single analog-to-digital converter (ADC) and the sequence is recovered by FHT processing and demodulating.

We analyze AC and DC-biased O-OFDM systems in AWGN channel by adding a Gaussian noise source in the electrical domain, as indicated in Fig. 1. Robustness against clipping is a crucial issue in IM/DD systems. Thus, additional noise due to the clipping effect is considered. The total noise is the sum of two independent contributions, i.e. the Gaussian and clipping noise, whose distribution is derived in [8]. In DC-biased O-OFDM (DCO-OFDM), the signal to be transmitted is $s(t) = h(t) + B_{DC}$, where residual negative peaks are clipped. With a suitable choice of the bias value B_{DC}, the clipping noise can be negligible. This is at the expense of the system power efficiency. When AC is applied, $s(t)$ is simply given by clipping the signal $h(t)$ at zero. If only the odd-indexed IDHT inputs support data, all the clipping noise falls into the even-indexed DHT outputs without affecting the data recovery.

III. Performance Analysis

To evaluate the performance of the proposed system, we compare it with AC and DCO-OFDM based on FFT, proposed in previous work [9]. To do that, we assume $r(t) = s(t) + n(t)$ (see Fig. 1). We compare DMT systems based on N-order FFT and FHT able to transmit the same data rate per parallel processing. Since FHT supports the double of constellation symbols, BPSK, 4, 8, 16 and 32 PAM are required to transmit at the same bit rate of FFT-based O-OFDM using 4, 16, 64, 256 and 1024 QAM, respectively.

We first consider O-OFDM using AC (ACO-OFDM). The performance in AWGN channel is reported in Fig. 2. The BER

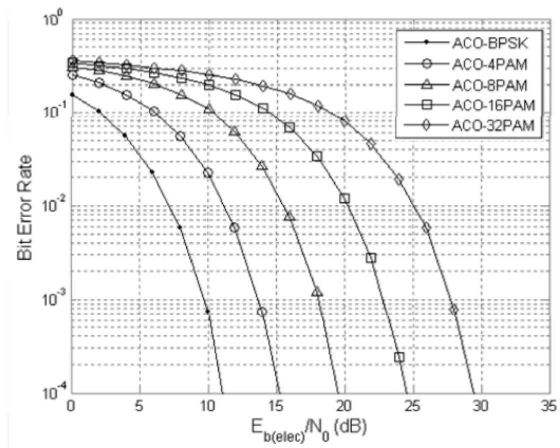

Fig. 2. BER versus normalized bit electrical energy for DHT-based ACO-OFDM using different real constellations (BPSK, 4, 8, 16, 32 PAM).

curves perfectly match the ones obtained by FFT modulation using complex, larger-size constellations [9]. This can be explained as the modulation is based on trigonometric transforms whose kernels only differ for the imaginary unit: the real and imaginary parts of the DFT equal the even and negative odd parts of the DHT, respectively [5]. Thus, by means of mirror-symmetric subcarriers, the DHT allows transmitting an information bit sequence mapped into a real constellation with similar performance of real-valued DFT processing, where the data sequence is mapped into a half-length vector of QAM symbols.

The bit electrical energy normalized to the noise power spectral density ($E_{b(elec)}/N_0$) increases with the constellation size. The electrical energy can be easily related to the optical power, if $N \geq 64$ [5], [9]

$$E_{b(opt)}/N_0 = E_{b(elec)}/N_0 + 10log_{10}(1/\pi). \quad (2)$$

We analyze the clipping effect impact on DCO systems, for different clipping level by varying the constellation size. Modulation formats from BPSK to 32 PAM are considered for 7dB and 13dB bias levels, in order to compare our system with [2] and [9]. The bias level in dB is measured as $10log_{10}(k^2+1)$ and it is related to the bias value $B_{DC} = k\sqrt{E\{h^2(t)\}}$, where $E\{h^2(t)\}$ is the signal variance [9].

The minimum bias is twice the signal standard deviation and the largest bias is optimum for a maximum modulation format of 256 QAM [2]. Figure 3 reports the BER curves versus the $E_{b(elec)}/N_0$ of DCO-OFDM using DHT. Again, good

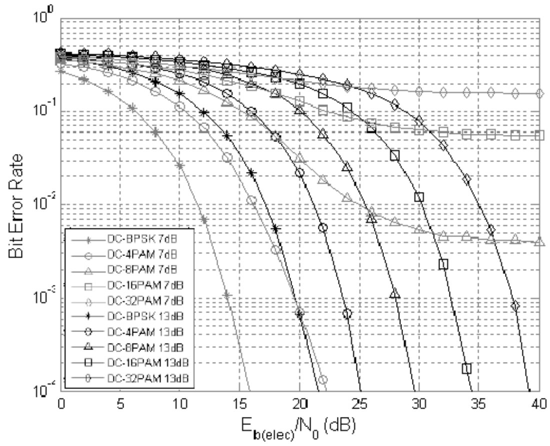

Fig. 3. BER versus $E_{b(elec)}/N_0$ for DHT-based DCO-OFDM with 7 and 13dB bias.

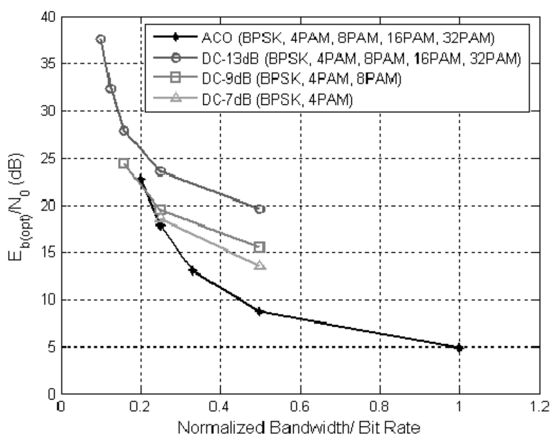

Fig. 4. $E_{b(opt)}/N_0$ versus normalized bandwidth at 10^{-3} BER for DHT-based AC and DC-biased (with 7, 9 and 13dB) O-OFDM. Each point indicates a constellation size (the smaller the size, the larger the normalized bandwidth).

matching with the BER curves of FFT-based OFDM [9] using larger constellations (from 4 to 256 QAM) is observed. The bias choice has the same impact on the system performance. For large constellations, 7dB bias results in a BER floor. In order to obtain acceptable BER, the bias has to be increased according to the modulation format. A BER of 10^{-3} ensures error free transmission, if forward error correction is applied. Figure 3 shows that with 13dB bias, clipping noise is still negligible when 32 PAM is used.

Using 9dB bias level, that can be considered optimum for 64 QAM modulation format [1], we have found that the clipping noise limits the constellation size of DHT-based O-OFDM to 8 PAM, which is the corresponding real constellation size to transmit at the same bit rate as a 64 QAM FFT-based system ($M = \sqrt{64} = 8$). For higher-order modulation formats, BER floor occurs above 10^{-3}, thus a larger bias is needed. If DHT-based O-OFDM is combined with adaptive modulation [2], different transform inputs support different modulation formats. A fixed large bias must be used, according to the maximum constellation size supported. The normalized bit electrical energy increases with the bias value. The bit optical power also increases and can be theoretically derived according to formula (4) in [9], which holds for both DFT and DHT modulation, as demonstrated in [5]

$$E_{b(opt)}/N_0 = E_{b(elec)}/N_0 + 10log_{10}(k^2/(k^2+1)). \qquad (3)$$

Figure 4 shows the performance of the different solutions in terms of optical power efficiency versus normalized bandwidth per bit rate at a target BER of 10^{-3}. The normalized bandwidth is defined according to [9] and takes into account that ACO-OFDM uses only half of the available transform points for data. Therefore, to transmit with the same spectral efficiency as DCO using M-PAM, ACO requires M^2-PAM. However, as shown in Fig. 4, for normalized bandwidths greater than 0.2, ACO-OFDM presents the best performance resulting more power efficient than any DC-biased system. For example, to achieve 10^{-3} BER at 0.5 normalized bandwidth, DCO with BPSK and minimum bias requires $4.7dB$ more power than ACO using 4 PAM. Below the value 0.2, ACO would require more power than DCO and very large size modulation formats. The highest spectral efficiency (corresponding to 0.1 normalized bandwidth/bit rate) is achieved with 13dB DCO at the expense of the system power efficiency: the required $E_{b(opt)}/N_0$ is larger than $35dB$. It is important to note that, due to the bias dependence on the constellation size, the DCO solution is less suitable than AC to adaptive scheme.

IV. Conclusion

In this letter, the same performance as FFT-based O-OFDM using complex constellations from 4 QAM to 1024 QAM is obtained with FHT real processing and using constellations from BPSK to 32 PAM. HS is not required and the system implementation has reduced complexity. Simulation results evidence that the bias choice has the same impact on the performance of FFT- and FHT-based systems. As DC-biased option limits the performance of adaptively modulated O-OFDM in terms of flexibility and power efficiency, AC using FHT, thanks to its simplified scheme, is an attractive solution for designing novel adaptive high-speed IM/DD systems.

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