MIMO-OFDM PAPR Reduction by Residue Number System

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***Abstract*-Multi-input multi-output orthogonal frequency division multiplexing (MIMO-OFDM) system has been widely accepted a promising scheme for wireless communication systems. However it still suffers the high peak-to-average power ratio (PAPR), which is the main limitation of OFDM-based systems. In this paper, we present a residue number system (RNS) based PAPR reduction scheme in MIMO-OFDM systems. The proposed scheme makes use of the properties of RNS to greatly reduce the PAPR and the computational complexity as well. Compared with the partial transmit sequence (PTS) scheme, the RNS-based PAPR reduction scheme has not only much better PAPR reduction performance without restriction to modulation format, but also low computational complexity without side information.**

***Keywords*-RNS, PAPR, PTS, MIMO**

I. INTRODUCTION

One of the main challenges of OFDM-based systems is the high peak-to-average power ratio (PAPR) of transmitted signals, resulting in signal distortion. The combination of multi-input multi-output (MIMO) and orthogonal frequency division multiplexing (OFDM) could exploit the spatial dimension capability to improve the system capacity by employing spatially separated antennas. In MIMO-OFDM systems, independent OFDM signals are transmitted from multiple transmit antennas. Therefore, MIMO-OFDM systems still suffer an inherent drawback of high PAPR.

There are some limitations for lossy PAPR reduction technologies, such as clipping, peak windowing, companding transform, etc [1], [2]. Nonlinear distortion and clipping of the transmitted signals lead to system's performance degradation. Recently several lossless PAPR reduction technologies have been proposed and investigated [3], [4], [5], [6], [7]; among them, the partial transmit sequence (PTS) scheme is an efficient approach and a lossless scheme for PAPR reduction by optimally combining signal sub-blocks. Selective mapping (SLM) is also a good approach, in which some statistically independent sequences are generated from the same information and the sequence with the lowest PAPR is transmitted. Both schemes provide improved PAPR statistic at the cost of additional complexity and loss of the data rate, because they need to implement some extra IFFT and iterations of phase optimization and transmit the side information. In addition, SLM scheme leads to a higher computational complexity at the same level of PAPR reduction, because it operates on all carriers [8], [9].

Residue number system (RNS), a parallel number system, is based on Chinese remainder theorem (CRT), which divides a large integer into several independent and parallel smaller ones with a specific modulus set. Due to the carry-free and parallel properties, RNS further simplifies the computations by decomposing a problem into a set of parallel, independent residue computations [10]. Recently, more attention is also paid to RNS in parallel communication field because of its parallel and fault-tolerant properties [11], [12]. An RNS-based OFDM transmission was proposed in [2], where we concentrated on the RNS-based OFDM system's description and on the PAPR simulation results.

In MIMO-OFDM, a new kind of PAPR reduction scheme by RNS will be presented in this paper. We utilize the parallel property of RNS to convert input signals into smaller residue signals, which are transmitted in a set of parallel, independent residue sub-channels; and make use of the characteristic of RNS modular operation to effectively reduce the PAPR. We will evaluate its performance in comparison with conventional MIMO-OFDM and PTS-MIMO-OFDM. It is demonstrated that the proposed scheme improves PAPR performance and greatly reduces computational complexity.

This paper is organized as follows: Section II gives an overview of PAPR and PTS in MIMO-OFDM. The proposed PAPR reduction scheme is described in Section III. Then we evaluate the performance of PAPR reduction and computational complexity in Section IV, while the conclusions are offered in Section V.

II. SYSTEM MODEL

Throughout the paper we consider N_T transmit antennas, over which independent data streams should be communicated. Space time coding as, e.g., in [7] is not considered here.

A. PAPR of MIMO-OFDM

The PAPR of output signals at each antenna is defined as the ratio between the maximum peak power and the average power

$$PAPR_{n_t} = 10\log\frac{\max\{|s_{n_t,k}|^2\}}{E\{|s_{n_t,k}|^2\}}(dB) \qquad (1)$$
$$(n_t = 1,2,\ldots N_T; k = 0,1,2\ldots, N-1)$$

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In MIMO-OFDM, the PAPR of all N_T transmit signals should be simultaneously as small as possible, which is defined as

$$PAPR = \max\{PAPR_1, PAPR_2, \ldots PAPR_{N_T}\} \quad (2)$$

It is known that the CCDF (Complementary Cumulative Distribution Function) is commonly used to denote the probability that the PAPR exceeds a given threshold value z, for conventional OFDM as shown in (3).

$$P\{PAPR > z\} = 1 - \{PAPR \le z\} = 1 - (1 - e^{-z})^N \quad (3)$$

In MIMO-OFDM, since the N_T number of antennas, the CCDF is presented

$$P\{PAPR > z\} = 1 - \{PAPR \le z\} = 1 - (1 - e^{-z})^{N_T N} \quad (4)$$

It can be seen from (3) and (4) that the PAPR performance of MIMO-OFDM systems is even worse than that of OFDM.

B. Partial Transmit Sequence in MIMO-OFDM

The block diagram of partial transmit sequence (PTS) scheme in MIMO-OFDM is shown in Fig.1. In each antenna channel it is a single antenna PTS-OFDM. It partitions an input data block of N symbols into M disjoint sub-blocks as follows:

$$X = [X^0, X^1, \ldots X^{M-1}]^T \quad (5)$$

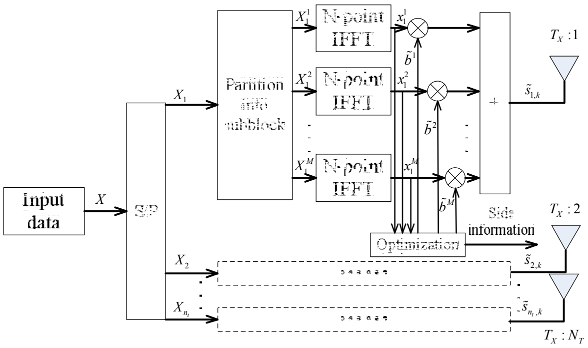

Fig.1 The block diagram of PTS scheme in MIMO-OFDM

Then each partitioned sub-block is multiplied by a complex phase factor $b^\mu = e^{j\phi\mu}$, $\mu = 1, 2, \ldots, M$,subsequently taking its IFFT to yield

$$x = IFFT\{\sum_{\mu=1}^{M} b^\mu X^\mu\} = \sum_{\mu=1}^{M} b^\mu x^\mu \quad (6)$$

After the PAPR comparisons among the candidate sequences, the optimal phase factor $\tilde{b}^\mu$ can be got. And the corresponding signal in the n_t antenna with the lowest PAPR can be expressed as

$$\tilde{s}_{n_t,k} = \sum_{\mu=1}^{M} \tilde{b}^\mu x^\mu, 0 \le k \le N-1, 1 \le n_t \le N_T \quad (7)$$

III. RNS-BASED PAPR REDUCTION

An RNS is defined by the relative prime modulus set $m_v (v = 1, 2, \ldots V)$. Any integer R can be represented in RNS by residue sequence $\{r_1, r_2, \cdots, r_V\}$.

$$r_v \equiv R (\bmod\, m_v) \quad (8)$$

The number r_v is said to be the residue of R with respect to m_v, and we shall usually denote this by $r_v = \langle R \rangle_{m_v}$. In this sense, a big integer can be converted into the small residues in RNS, and these residues are always smaller than the corresponding modulus. The integers in the range of $[0, M_I)$ can be represented in this RNS uniquely and unambiguously, where $M_I = \prod_{i=1}^{V} m_v$ is referred to as the information dynamic range, i.e., the legitimate range of the information symbol.

The information symbols can be uniquely recovered by residue sequence through CRT, which is one of the fundamental theorems of RNS. The relationship between the information symbols R and its residues is as follows

$$R = \left(\sum_{v}^{V} S_v \cdot \langle 1/S_v \rangle_{m_v} \cdot r_v \right) \bmod M_I \quad (9)$$

where $\langle 1/S_v \rangle_{m_v}$ called as multiplicative inverse of S_v , $S_v = M_I / m_v$ and $(S_v \cdot \langle 1/S_v \rangle_{m_v}) \bmod m_v = 1$.

The definition of signed number in RNS is similar to that in TCS (Two's Complement System) [10], [13]. An integer R in the legitimate range $[0, M_I)$ can be represented as a signed number, $\tilde{R}$ .Then if $0 \le R < \lceil M_I / 2 \rceil$ or $\lceil M_I / 2 \rceil \le R < M_I$, $\tilde{R}$ is positive and negative respectively, where $\lceil x \rceil$ denotes the smallest integer larger than x .

The basic diagram of RNS-based PAPR reduction scheme in MIMO-OFDM is given in Fig.2. The number of modulus $\{m_1, m_2, \cdots m_V\}$ is V , and the input are converted into V residues by the corresponding modulus set, and the number of transmit antennas equals the number of residue sub-channels. These residue signals are preformed OFDM modulation in the corresponding residue channels. In the each of the V parallel residue sub-channels one IFFT of length N is employed.

The function of mapping module, if the input is positive, it can be sent into B/R (binary to residue) module directly; otherwise the input adds the legitimate M_I before B/R.

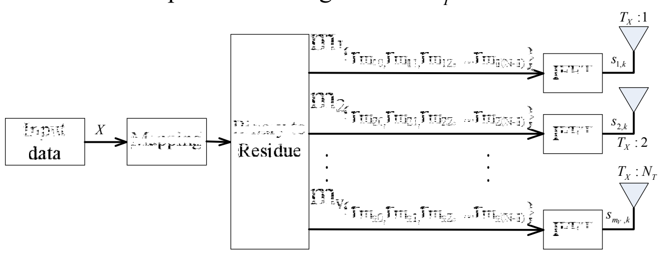

Fig.2 The basic diagram of RNS-based scheme in MIMO-OFDM

Through B/R conversion, according to (8), the serial data streams are divided into V parallel residue sub-channels transmitting signals.

In each residue sub-channel, the residue sequences $\{r_{m_v 0}, r_{m_v 1}, \ldots, r_{m_v (N-1)}\}$ which correspond to the modulus m_v residue sub-channel are transmitted into IFFT module respectively. The output corresponding to the modulus m_v residue sub-channel after IFFT is represented as follows.

$$s_{m_v,k} = s(kT/N) = \sum_{i=0}^{N-1} r_{m_v,i} \exp(j\frac{2\pi ik}{N}) \quad (0 \le k \le N-1, 0 \le i \le N-1) \tag{10}$$

A. PAPR of RNS-based scheme

The real and imaginary parts of OFDM signals have asymptotically Gaussian distributions for a large number of subcarriers by the central limit theorem. Then the amplitude of the OFDM signals follows a Rayleigh distribution. The PAPR of RNS-based scheme in each sub-channel can be written as

$$PAPR_{n_t} = 10\log\frac{\max\{\left|\sum_{i=0}^{N-1} r_{m_v,i}\exp(j\frac{2\pi ik}{N})\right|^2\}}{E\{\left|\sum_{i=0}^{N-1} r_{m_v,i}\exp(j\frac{2\pi ik}{N})\right|^2\}} = 10\log\frac{\max\{\left|\sum_{i=0}^{N-1} r_{m_v,i}\exp(j\frac{2\pi ik}{N})\right|^2\}}{2\sigma^2}(dB) \tag{11}$$

where σ is the variance of OFDM signals. In the MIMO-OFDM sceneries, the PAPR performance is governed by the worst-case PAPR, it can be presented as

$$PAPR_{rns-mimo} = \max_{n_t=1,2,\dots N_T} PAPR_{n_t} = 10\log\frac{\max_{n_t}\{\left|\sum_{i=0}^{N-1} r_{m_v,i}\exp(j\frac{2\pi ik}{N})\right|^2\}}{2\sigma^2}(dB) \tag{12}$$

According to (8), the residue is always smaller than the corresponding modulus, which may be chosen smaller than the original number. Then the residue is smaller than the original number. After multiplying a rotation factor and summing up all the N elements, it is still smaller than the sum of original one. It can be seen that the proposed scheme has the potential to improve the PAPR reduction performance.

B. Complexity

In RNS, the addition and multiplication are modular operations. In theoretical analysis, they can be designed for flexibility in which case the methodology allows the design of adders for any modulus. The basic adder for any modulo-m is defined as (13)

$$\langle A+B\rangle_m = \begin{cases} A+B & \text{if } A+B < m \\ A+B-m & \text{otherwise} \end{cases} \tag{13}$$

In the most straightforward implementation, the most complex way, a basic modular requires 3 adders: one for the addition, one for the subtraction, and one for the comparison [13].

A modular multiplication of complex signals can be expressed as (14)

$$\langle A\times B\rangle_m = \langle\langle a_1a_2\rangle_m - \langle b_1b_2\rangle_m\rangle_m + i\langle\langle b_1a_2\rangle_m + \langle b_2a_1\rangle_m\rangle_m \tag{14}$$

The modular multiplication of complex signals needs more 6 modular operations than complex multiplier. In each modular operation, it needs 2 adders (one for addition and one for comparison), which is similar to the case of the modular adder.

Based on the definition of RNS, the residue is smaller than its corresponding modulus, i.e. $0 \le r_v < m_v, (1 \le v \le V)$. Regardless of the number of addition and multiplication, the sum of residue signals in each residue sub-channel is still smaller than its corresponding modulus. It can be seen that this scheme effectively controls the dynamic range of the transmitted signals to improve the PAPR reduction performance.

IV. SIMULATION RESULTS

In this section, some simulations are employed to demonstrate PAPR reduction performance and computational complexity comparison between the proposed scheme and the original PTS scheme. The OFDM symbol of each antenna channel contains 2048 subcarriers, and for simplicity, we expect all N sub-carriers to be active.

A. Complexity Analysis

In this part, the overall computational complexity of RNS scheme in MIMO-OFDM will be discussed.

A complex complication takes 4 real multiplications and 2 real additions, and a complex addition requires 2 real additions. Furthermore, it can be assumed that the complexity of a real multiplication equal the complexity of 4 real additions [8]. In the RNS scheme according to (10), it needs the number of modulus V N-pointed IFFT operations. Considered the input as the complex signal, a modular addition would take 6 real additions in the most complexity situation and a modular multiplication would take 30 real additions. $\left|s_{n_t,k}\right|^2$ is calculated to determine the PAPR, which requires $2VN$ real multiplications and VN real additions.

In general, a length N IFFT operation requires $(N/2)\log_2 N$ complex multiplications and $N\log_2 N$ complex additions. So the overall computational complexity of RNS and PTS scheme [9] can be summarized through real additions in Table I.

TABLE I COMPUTATIONAL COMPLEXITY OF RNS AND PTS BY THE NUMBER OF EQUIVALENT REAL ADDITIONS

PAPR reduction schemes	PTS	RNS
IFFT	$11N_TMN\log_2 N$	$21VN\log_2 N$
Addition of all phrase factors	$2N_T(M-1)W^{M-1}N$	-
PAPR computation	$9N_TW^{M-1}N$	$9VN$
Equivalent real additions	$N_T[11MN\log_2 N + (2M+7)W^{M-1}N]$	$21VN\log_2 N + 9VN$
$V = N_T = 3$, $N = 2048$, $W = 2$	M=3: 2549760 M=8: 24035328	1474560

As to computational complexity, compared with PTS, the RNS-based PAPR reduction scheme in MIMO-OFDM can result in 42.2% and 93.9% reduction in equivalent real additions for M=3 and M=8, respectively.

Note that in PTS scheme the complexity of searching increases exponentially with the number of sub-blocks. In the comparison, the implementation of RNS-based PAPR reduction scheme is supposed in the most complex way. However, the binary phase factors of {1,-1} are used, i.e. W=2, the computational complexity of the rotation of each sub-block for the PTS scheme is reduced. The proposed scheme has the potential to reduce the computational complexity compared with PTS-MIMO-OFDM scheme.

B. *PAPR Reduction*

The performance of PAPR reduction is evaluated by CCDF. To compare with the original PTS scheme in MIMO-OFDM, we assume the antenna number of two schemes and the subcarrier number in each sub-channel are the same. Each OFDM symbol contains $N = 2048$ subcarriers throughout, where the number of input symbols is 1000. The parameter used for simulation is shown in Table II.

TABLE II PARAMETER USED FOR SIMULATION OF PAPR

Parameters	Value
Subcarrier number, N	2048
The number of input symbols	1000
Antenna number, N_T	3
Modulation format	64QAM/4QAM
Moduli number of RNS, V	3
Moduli set of RNS	{128,127,63}
PTS Sub-block number, M	3/8
PTS phrase factor	{1,-1}

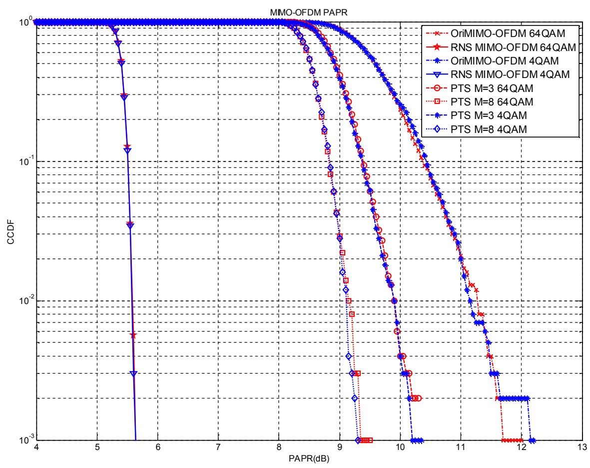

Fig.3 PAPR reduction performance of the proposed scheme, PTS scheme and the conventional MIMO-OFDM with N=2048, 4QAM/64QAM V=3, W={-1,1}, M=3/M=8.

Fig.3 compares the PAPR reduction performance of the proposed scheme, the PTS scheme and the conventional MIMO-OFDM, where the modulation style is 64QAM and 4QAM respectively. The curves labeled by "RNS MIMO-OFDM 64QAM", "RNS MIMO-OFDM 4QAM" denote the PAPR performance of RNS-based scheme. The curves labeled by "PTS M=3 4/64QAM","PTS M=8 4/64QAM" denote the PAPR performance of the PTS scheme in MIMO-OFDM with $M = 3$ and $M = 8$ disjoint sub-blocks respectively. When $M = 3$, $W = \{-1,1\}$, the RNS-based scheme is better than the PTS by about 5dB. When $M = 8$, the proposed scheme still outperform PTS. Meanwhile, the computational complexity of the proposed scheme reduces to just 6.1% of that of the PTS. The curves labeled by "Ori-MIMO-OFDM 64QAM/4QAM" denote the conventional MIMO-OFDM PAPR performance. About 6dB improvement of PAPR reduction is obtained by the proposed scheme, at the CCDF of 10^{-3}.

In addition, nearly no differences between the CCDF for 4QAM, 64QAM signaling are visible. The proposed scheme is not restricted to modulation format.

V. CONCLUSIONS

An RNS-based PAPR reduction scheme in MIMO-OFDM is presented in this paper, which utilize the properties of RNS and characteristic of RNS modular operation to effectively reduce the PAPR without side information. Theoretical analysis and simulation results demonstrate the proposed scheme outperforms the PTS scheme in the PAPR reduction performance and the computational complexity.

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