Weighted Cyclic Prefix OFDM: PAPR Analysis and Performances Comparison with DFT-Precoding

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Abstract—In this paper, we present a weighted cyclic prefix orthogonal frequency-division multiplexing (WCP-OFDM) transceiver as a generalization of traditional cyclic prefix (CP)-OFDM. In time-variant channels, this multicarrier transmission scheme may mitigate inter-channel interference (ICI) thanks to the use of non-rectangular pulse shapes. A precoding step may be required in order to reduce the peak-to-average power ratio (PAPR) at the transmitter output. For instance, a discrete Fourier transform (DFT) precoder leads to a single carrier transmission scheme with frequency domain equalization. We analyze the consequences of such a precoding, in terms of performances, in the context of a time-frequency selective channel.

I. INTRODUCTION

Cyclic prefix orthogonal frequency-division multiplexing (CP-OFDM) diagonalizes multipath time-invariant channels if the guard interval is longer than the channel impulse response. As a consequence, perfect reconstruction may be obtained by means of a single-tap per sub-channel equalizer [1]. Furthermore, the fast Fourier transform (FFT) algorithm ensures a low-complexity implementation. However, the Doppler spread introduced by time-variant channels breaks the orthogonality between sub-channels and inter-carrier interference (ICI) appears [2, p. 753].

Filter bank based multicarrier (FBMC) systems offer a more general transmission framework, providing non-rectangular pulse-shape. The purpose of this approach is to design matched prototype filters to time-frequency selective channels in order to yield better performance results [3]–[5]. Despite attractive performances, FBMC systems are rarely recommended in standardized applications because of their computational complexity. Indeed, they require the use of polyphase matrix filtering whose complexity increases with the length of the prototype filters [6].

In this work, we focus on short prototype filters that ensure a low-complexity implementation. Thus, we consider here the case of filters with the same length as rectangular pulses used in CP-OFDM. The difference with CP-OFDM is that the pulses do not need to be rectangular. Such a generalization of CP-OFDM is referred to as weighted cyclic prefix (WCP)-OFDM.

As a multicarrier transmission scheme, WCP-OFDM is characterized by its high peak-to-average power ratio (PAPR) wich is a major drawback for its hardware implementation.

In this study, we compare the PAPR of CP-OFDM with a time-frequency optimized WCP-OFDM. We also define a DFT-precoding block which leads to a single carrier (SC) block transmission scheme with frequency domain equalization. We compare the performances of both systems with LDPC channel coding and assuming a time-frequency selective channel.

II. WEIGHTED CYCLIC PREFIX OFDM TRANSCEIVER: STRUCTURE AND PERFECT RECONSTRUCTION CONDITIONS

We want to transmit a sequence of complex symbols denoted $\{c_{m,n}\}_{(m,n)\in\Lambda}$ where $\Lambda = \{0,\ldots,M-1\}\times\mathbf{Z}$. The symbols are assumed to be independent and identically distributed. Their mean power is denoted $\sigma_c^2 = \mathrm{E}\{|c_{m,n}|^2\}$.

Each $c_{m,n}$ is placed in the time-frequency plane at coordinates $(m/M, nN)$ where M is the number of sub-channels and N represents the number of sample per sub-channel symbol period. The oversampling ratio is given by N/M. The transmitted signal can be written

$$s[k] = \sum_{(m,n)\in\Lambda} c_{m,n}\gamma_{m,n}[k], \qquad k \in \mathbf{Z} \qquad (1)$$

where $\gamma_{m,n}[k]$ is a time-frequency shifted version of the prototype pulse $\gamma[k]$ defined as

$$\gamma_{m,n}[k] = \frac{1}{\sqrt{M}}\gamma[k-nN]e^{j2\pi\frac{m}{M}k}, \qquad \gamma[k] \in \mathbf{R}. \qquad (2)$$

And the complex symbols can be estimated thanks to a projection over a demodulation basis $\check{\gamma}_{m,n}[k] = \frac{1}{\sqrt{M}}\check{\gamma}[k-nN]e^{j2\pi\frac{m}{M}k}$, with $\check{\gamma}[k] \in \mathbf{R}$:

$$\hat{c}_{p,q} = \sum_{k=qN}^{(q+1)N-1} r[k]\check{\gamma}_{p,q}^*[k] \qquad (3)$$

$$= \frac{1}{\sqrt{M}} \sum_{k=qN}^{(q+1)N-1} r[k]\check{\gamma}[k-qN]e^{-j2\pi\frac{p}{M}k} \qquad (4)$$

with $r[k]$ the received signal.

The resulting transmission scheme can be efficiently realized thanks to the use of fast alogrithms [7]. This leads to the scheme depicted in fig. 1 in which we have added an optional precoding step (Q block). Thus, this generalizes the CP-OFDM
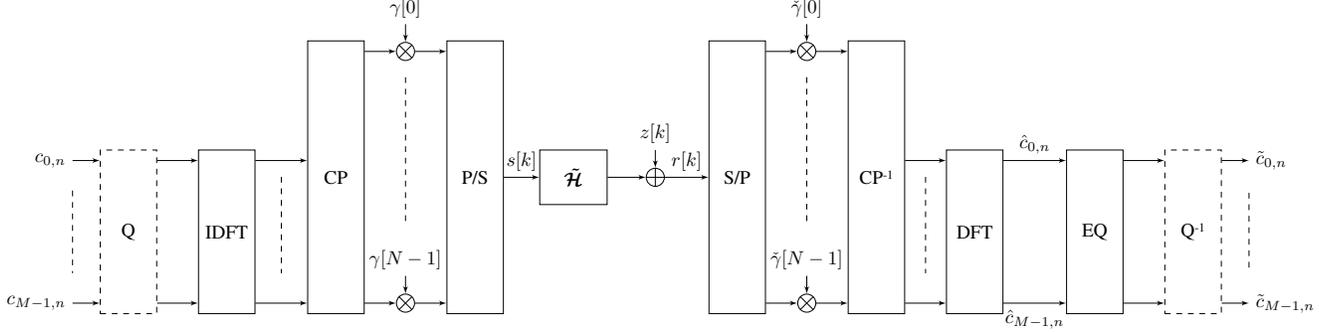
Fig. 1: Efficient implementation of a precoded WCP-OFDM transmultiplexer.

transceiver by allowing non-rectangular pulse shapes while preserving a low-complexity.

This transmission scheme is thus referred to as weighted cyclic prefix (WCP)-OFDM. The transmission process consists in a projection of M complex symbols over an exponential basis (IDFT block) after an optional precoding step (Q block). This projection is followed by a cyclic extension of the data block (CP block) defined by the following: if we denote $x[k]$ the entries of CP block, $0 \leq k \leq M - 1$, and $y[k]$, $0 \leq k \leq N - 1$, its outputs, then we have $y[k] = x[2M - N + k]$ for $0 \leq k \leq N - M - 1$ and $y[k] = x[M - N + k]$ for $N - M \leq k \leq N - 1$.

The resulting N samples are weighted by $\gamma[k]$, $0 \leq k \leq N - 1$. At the receiver side, the dual operation is performed in order to ensure perfect reconstruction (PR) of the transmitted symbols, that is to say $\hat{c}_{m,n} = c_{m,n}$ if $r[k] = s[k]$. In particular, CP^{-1} block is defined by the following: if we denote $x[k]$ the entries of the CP^{-1} block, $0 \leq k \leq N - 1$, and $y[k]$, $0 \leq k \leq M - 1$, its outputs, then we have $y[k] = x[N - M + k]$ for $0 \leq k \leq 2M - N - 1$ and $y[k] = x[k - (2M - N)] + x[p - (M - N)]$ for $2M - N \leq k \leq M - 1$.

As shown in [7], if the received signal $r[k]$ is equal to the transmitted signal $s[k]$, then the complex symbols $\{c_{m,n}\}_{(m,n) \in \Lambda}$ can be exactly reconstructed provided that the following Perfect Reconstruction (PR) conditions are fulfilled:

- $\gamma[k]\check{\gamma}[k] + \gamma[k+M]\check{\gamma}[k+M] = 1$ for $0 \leq k \leq N - M - 1$,
- $\gamma[k]\check{\gamma}[k] = 1$ for $N - M \leq k \leq N - 1$.

Through this relation, one may recover the expression of rectangular filters used for CP-OFDM: if $\gamma[k] = 1$ for $0 \leq k \leq N - 1$ and 0 otherwise then PR conditions are satisfied when $\check{\gamma}[k] = 1$ for $N - M \leq k \leq N - 1$ and 0 otherwise.

Apart from the cyclic prefix case, PR conditions may be particularized in the linear phase orthogonal case, such that $\gamma[k] = \check{\gamma}[k] = \gamma[N - 1 - k]$ [2, p. 160]. In this context, the work presented in [8] provides closed-form expressions for optimized prototypes filters. Optimization criteria include time-frequency localization (TFL) maximization (fig. 2).

We have shown in [9] that the TFL filter discussed above outperforms CP-OFDM in several multipath time-variant environments, using a single-tap per sub-channel equalizer. One may notice that WCP-OFDM offers a good trade-off between
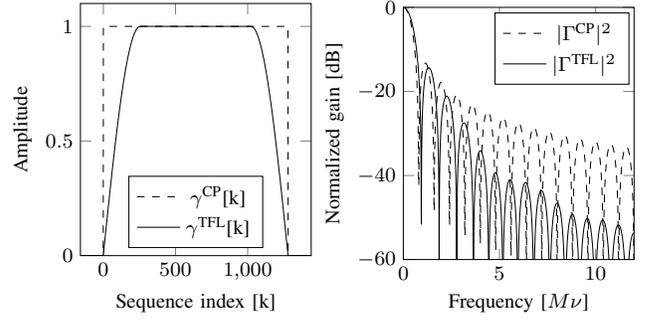

Fig. 2: Time and frequency responses of the prototype filters with $\Gamma(\nu) = 1/\|\gamma\|_{\infty} \sum_{k=0}^{N-1} \gamma[k] \exp(-j2\pi\nu k/N)$, using $M = 1024$ and $N = 1280$.

complexity[1] and ICI mitigation, unlike more general filtered multitone setups [10].

III. WCP-OFDM AND PAPR ISSUE

Multicarrier modulations are characterized by their high peak-to-average power ratio (PAPR) defined by

$$\text{PAPR} = \frac{\max_{0 \leq t \leq T_0} \{|s(t)|^2\}}{\frac{1}{T_0} \int_0^{T_0} |s(t)|^2 \, dt} \tag{5}$$

where $T_0 = N/B$ and $s(t) = \sum_k s[k]\text{sinc}(Bt - k)$ with B representing the transmission bandwidth. Linear power amplification is difficult to achieve in presence of high PAPR. As a consequence, the clipping probability may be estimated thanks to the PAPR complementary cumulative distribution function (CCDF): $F_c(\text{PAPR}_0) = \Pr\{\text{PAPR} > \text{PAPR}_0\}$, as presented in figure 3. Since the TFL pulse exhibits a non-constant impulse response, the overlying transmission scheme experiences a greater PAPR than traditional rectangular filters (CP) [11]. In both cases, PAPR is greater than 8 dB.

Many studies focus on PAPR reduction techniques for multicarrier modulations [12] (*e.g.* clipping and filtering, coding, tone reservation, tone injection, selected mapping, interleaving...). However, the techniques listed above may either distort the pulse shapes, require extra transmission power,

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[1]$2N$ additional multiplications are performed compared to CP-OFDM.
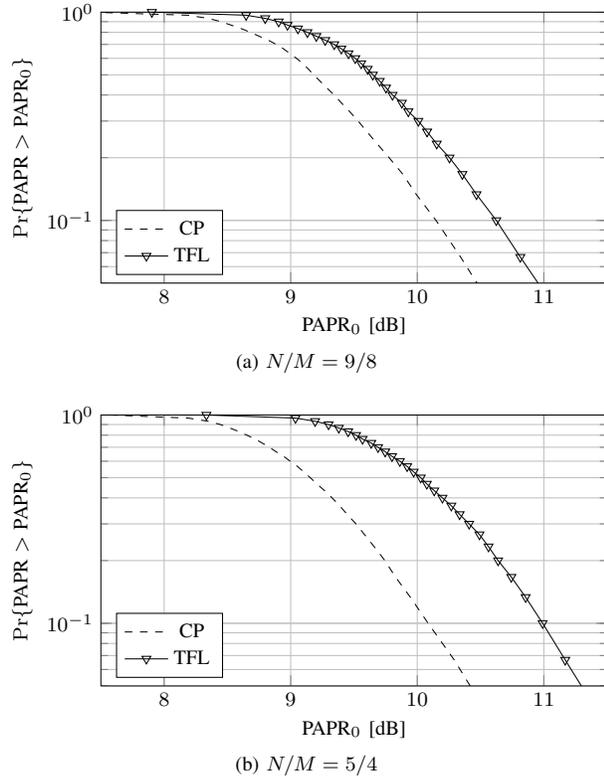
(a) $N/M = 9/8$

(b) $N/M = 5/4$

Fig. 3: PAPR CCDF for $M = 1024$ and $N/M \in \{9/8, 5/4\}$ for cyclic prefix and time-frequency localized pulses in a multicarrier transmission scheme.

decrease the spectral efficiency and they often bring prohibitive computational complexity.

A particular precoding technique consists in a simple discrete Fourier transform (DFT) at the transmitter side and an inverse DFT (IDFT) after the equalizer. It results in a single carrier transmission scheme with frequency domain equalization (SC-FDE). Indeed, DFT precoding followed by IDFT leads to an equivalent SC modulation without precoding nor IDFT. CP-OFDM and SC-FDE have been compared in several studies [13]–[16]. Obviously, it turns out that SC-FDE has a lower PAPR than CP-OFDM. However, multicarrier modulations allow a per sub-channel bit-loading and power allocation, leading to better throughput than single carrier modulations for a given bit-error-rate (BER). We also notice that CP-OFDM remains sensitive to frequency offsets such as Doppler shifts.

IV. SIMULATIONS : BER PERFORMANCE RESULTS

In this simulation framework, we consider a land mobile transmission system, using a band $B = 8$ MHz, centered around a frequency $f_c = 1$ GHz. We assume that the transmission takes place in an outdoor urban environment. Such a propagation may be fitted by a 6-path WSSUS channel model where the last path occurs at 5 μs (COST 207 TUx6 [17]) and whose impulse response is truncated to $L = 45$

coefficients. We develop here two mobility scenarios:

- low speed, with $v_{\max} = 3$ km/h ;
- high speed, with $v_{\max} = 350$ km/h.

At the transmitter side, the low-density parity check (LDPC) encoder uses 32400 bit codewords and operates at coding rate $R_c = 3/4$. Each codeword is randomly interleaved and split into 8 data blocks of $M = 2048$ quadrature phase shift keying (QPSK) symbols. The oversampling factor is set to $N/M = 5/4$ so that a transmitted block is made of $N = 2560$ symbols. We consider than each block is transmitted over independent channel realizations in order to ensure the best diversity scenario. In practice, symbols should be spaced by a period greater than the coherence time of the channel. In a similar way, sub-channels should be spaced by a band greater than the coherence bandwidth of the channel. Unfortunately, these requirements are usually difficult to achieve, considering various application constraints (*e.g.* spectral efficiency, interactivity, medium access control).

At the receiver side, a single-tap per sub-channel equalizer is used. After the deinterleaver, a LDPC decoder performs 10 iterations over each codeword without parity check[2]. We compare the performances of WCP-OFDM with or without DFT-precoding. In the first case, the transmission system leads to a single carrier (SC) block scheme. In the second case, the system leads to a regular multicarrier (OFDM). For both precoding strategies the transmitter and the receiver are provided with rectangular biorthogonal prototypes (CP) and time-frequency optimized orthogonal prototypes (TFL). Bit-error-rate (BER) is plotted as a function of E_b/N_0, where $E_b = N\sigma_c^2/2MB$ and $N_0 = \sigma_z^2/B$ (fig. 4).

First of all, we show that multicarrier schemes obtain better performances results than single carrier schemes (approximately a factor 5 for $E_b/N_0 = 10$ dB), for both low and high mobility scenarios. This result demonstrates that in presence of a random multipath channel, the OFDM technique is useful in order to circumvent the fading phenomenon to a limited number of sub-channels. On the contrary, if the channel experiences fading in a SC system, the whole block is usually lost.

We recall that the oversampling factor leads to $N - M > L$. As a consequence, cyclic prefix based transmission schemes do not introduce inter-block interference (IBI). However in the orthogonal scenario based on TFL prototypes, IBI remains present.

In the low mobility scenario, regardless of the precoding strategy, we show that the time-frequency localized pulse yields better results than the rectangular pulse. Since the TFL impulse response shows smooth transitions, it justifies a low IBI term and the large number of sub-channel ensures a sufficient partitioning of the band B. As a consequence, the overall interference term is negligible compared to the noise term and the performance curves (fig. 4a) reflect the need of orthogonal

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[2]This channel coding configuration ($R_c = 3/4$, 10 iterations) is chosen in order to achieve a statistically representative simulation with a reasonable computing time.

signaling in mainly noisy channels. Similar conclusions can be drawn in the high mobility scenario. However Doppler induced inter-carrier interference (ICI) appears and yield lower performances (fig. 4b).

For high values of E_b/N_0, the difference between the BER experienced by CP and TFL pulses tends to increase with the time selectivity of the channel. It confirms the interest of a time-localized pulse in such a scenario.
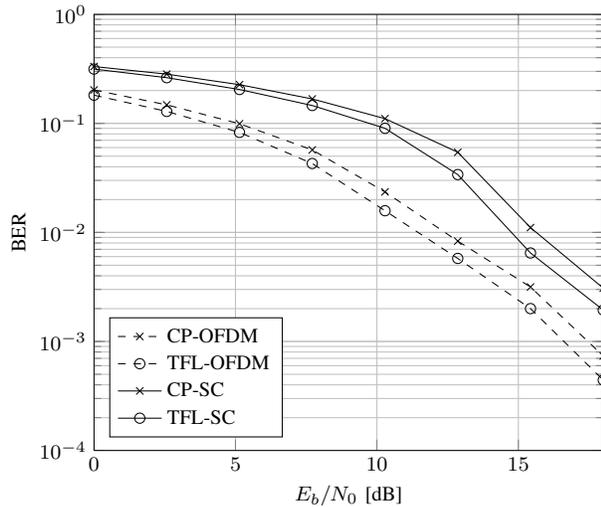
(a) BER performances in low mobility scenario.
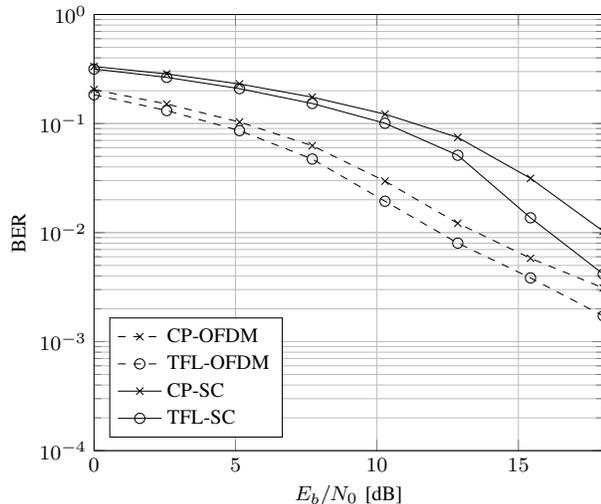
(b) BER performances in high mobility scenario.

Fig. 4: BER performances for two mobility scenarios in a COST 207 TUx6 channel. Comparison between single carrier and multicarrier schemes for $M = 2048$ and $N/M = 5/4$.

V. Conclusion

Weighted cyclic prefix OFDM systems generalize traditional CP-OFDM, allowing the use of non-rectangular filters. Even if time-frequency localized pulses yield interesting BER performances in time-frequency selective channels, they also introduce a greater PAPR than rectangular pulses, increasing with the oversampling factor. In order to mitigate the PAPR, a DFT-precoding may be used, leading to a single carrier block transmission scheme.

We have shown, through simulation, that best (LDPC coded) BER performance results are achieved by a multicarrier scheme, using TFL pulse, assuming a realistic time-frequency selective channel.

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