Linear Companding Transform for the Reduction of Peak-to-Average Power Ratio of OFDM Signals

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Abstract—**A major drawback of orthogonal frequency-division multiplexing (OFDM) signals is their high peak-to-average power ratio (PAPR), which causes serious degradation in performance when a nonlinear power amplifier (PA) is used. Companding transform (CT) is a well-known method to reduce PAPR without restrictions on system parameters such as number of subcarriers, frame format and constellation type. Recently, a linear nonsymmetrical companding transform (LNST) that has better performance than logarithmic-based transforms such as μ-law companding was proposed. In this paper, a new linear companding transform (LCT) with more design flexibility than LNST is proposed. Computer simulations show that the proposed transform has a better PAPR reduction and bit error rate (BER) performance than LNST with better power spectral density (PSD).**

Index Terms—**Companding transform (CT), linear companding transform (LCT), nonlinear power amplifier (PA), orthogonal frequency-division multiplexing (OFDM), peak-to-average power ratio (PAPR).**

I. Introduction

OFDM (orthogonal frequency-division multiplexing) is a multicarrier modulation scheme that divides the incoming bitstream into parallel, lower rate substreams and transmits them over orthogonal subcarriers. As a result, the bandwidth of each subcarrier is much smaller than ch333annel coherence bandwidth and hence each subcarrier will experience relatively a flat fade [1]. It is a bandwidth efficient modulation scheme and has the advantage of mitigating inter-symbol interference (ISI) in frequency selective fading channels. Today, OFDM is used in many wireless standards such as terrestrial digital video broadcasting (DVB-T), digital audio broadcasting (DAB-T), and has been implemented in wireless local area networks (WLANs) (IEEE 802.11a, ETSI Hiperlan2) and wireless metropolitan area networks (IEEE 802.16d).

The main drawback of OFDM is its high peak-to-average power ratio (PAPR) which causes serious degradation in performance when nonlinear power amplifier (PA) is used. This high PAPR forces the transmit PA to have a large input backoff (IBO) in order to ensure linear amplification of the signal, which significantly reduces the efficiency of the amplifier, Furthermore, high PAPR requires high resolution for the receiver analog-to-digital converter (A/D). Since the dynamic range of the signal is much larger for high PAPR, a high-resolution quantizer is required to reduce quantization error, which requires more bits and places a complexity and power burden on the receiver front end. In the literature, many solutions have been proposed to reduce PAPR such as block coding, selective mapping (SLM), partial transmit sequence (PTS), tone reservation and injection, [2 & reference therein]. However, most of these solutions have restrictions on system parameters such as number of subcarriers, frame format, and constellation type. Signal distortion solutions such as clipping [3], [4], and [21] and companding [5]–[17] can be used without restriction on the system parameters but at the price of increased bit error rate (BER) and spectral regrowth. Although clipping performs very well with low modulation orders, clipping error becomes very significant with higher orders and seriously degrades performance [17], [21], which makes companding more suitable for high data rates applications. The use of μ-law companding as PAPR reduction scheme for OFDM systems was firstly investigated in [5], where the authors presented an elegant theoretical performance analysis of companded OFDM signals. However, their work only considered the effect of quantization noise and ignored PA nonlinearity. In [12], a general companding transform was proposed, where the performance of four typical companding schemes; linear symmetrical transform (LST), linear nonsymmetrical transform (LNST), nonlinear symmetrical transform (NLST), and nonlinear nonsymmetrical transform (NLNST), were investigated. It was shown that, LNST is the best among the proposed companding schemes in terms of PAPR reduction and BER. These performance gains were achieved by introducing an inflexion point in LNST so that small and large signal amplitudes could be treated with different scales. This allows more flexibility and freedom in companding design to meet the system requirements such as PAPR reduction, required signal average power, Power amplifier characteristics, and BER. However, when the input signal passes through the inflexion threshold, transformed signal will have abrupt jump that degrades the power spectral density (PSD) of transformed signal. In [16], the authors proposed a linear transform that has one-to-one mapping between the input and the output transformed signals. The companding form was designed so that the output signal has no abrupt jumps, which resulted in a better PSD. However, its PAPR reduction capability and BER performance are lower than LNST. Furthermore, the effect of PA nonlinearity was ignored. In this paper, a new linear companding transform (LCT) is proposed, the proposed transform has two inflexion points to give more design flexibility. The performance of the proposed transform and LNST is evaluated in AWGN channel with the presence of nonlinear amplification by means of computer simulations.

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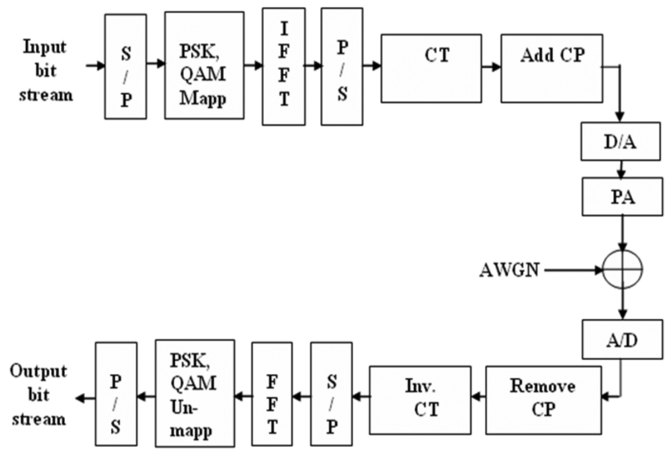

Fig. 1. Typical companded OFDM system.

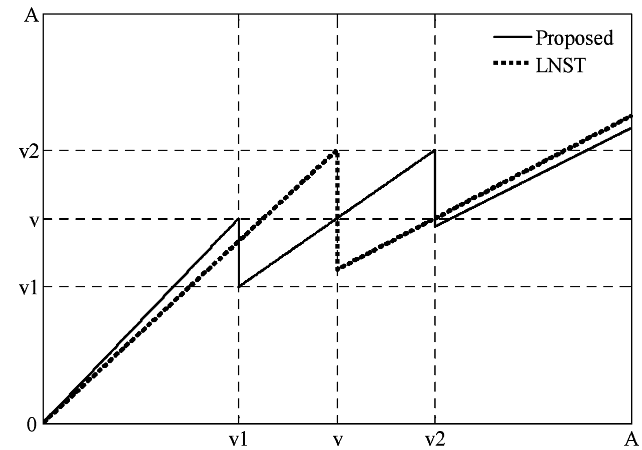

Fig. 2. Transforms profiles.

Results show that the proposed transform has a better PAPR reduction capability and BER performance than LNST with an enhanced PSD.

The rest of this paper is outlined as follows; Section II addresses the PAPR of OFDM signals. Section III introduces the proposed linear transform and discusses the design criteria. Section IV introduces the nonlinear power amplifier model that used in the simulation, while Section V discusses simulation results. The paper is concluded in Section VI. An analysis section is presented in Appendix A.

II. PAPR Formulation

Fig. 1 shows a typical companded OFDM system, where input bit stream is first converted into N parallel lower rate bit streams and then fed into symbol mapping to obtain symbols $[S_k = S_0, S_1, \ldots, S_{N-1}]$. These symbols are then applied to IFFT to generate OFDM symbol, which can be expressed as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k e^{j\frac{2\pi}{N}kn}, \qquad n = 0, 1, \ldots, N-1 \quad (1)$$

The PAPR of the discrete OFDM signal may be expressed as

$$PAPR = \frac{\max\{|x(n)|^2\}}{E\{|x(n)|^2\}} \quad (2)$$

If OFDM signal is oversampled by a factor ≥ 4, its PAPR is a good approximate to the one of continuous OFDM signal [18]. Oversampling by a factor of L can be achieved by padding the symbols S_k with $(L-1)N$ zeros. After IFFT, the resultant symbols are converted to serial and companding transform (CT) is performed. To guarantee that all transformed signals are under a given threshold, a digital clipping “not shown in Fig. 1” is used after the CT. Note that, due to the disadvantages of clipping, the CT should be designed cautiously so that the amount of clipped signals is as little as possible [12]. A cyclic prefix (CP) is then inserted to OFDM symbol interval to eliminate intersymbol interference (ISI).

III. Proposed Transform

It was shown in [12] that, a linear companding transform with an inflexion point (LNST) can outperform logarithmic-based companding transforms such as μ-law companding. LNST can be expressed as [12]

$$y(n) = \begin{cases} \frac{1}{u} \cdot x(n) & |x(n)| \leq v \\ u \cdot x(n) & |x(n)| > v \end{cases} \quad (3)$$

where $0 \leq u \leq 1$, and $0 \leq v \leq \max\{|x(n)|\}$. Since $x(n)$ is complex-valued, the companding transform should be applied to real and imaginary parts separately. At receiver, the original signal can be recovered according to

$$\begin{aligned} \tilde{x}(n) &= \begin{cases} uR(n) & n \in \varphi_1(v) \\ \frac{1}{u}R(n) & n \in \varphi_2(v) \end{cases} \\ R(n) &= y(n) + w(n) + q(n) \\ \varphi_1(v) &= \{n \forall |x(n)| \leq v\} \\ \varphi_2(v) &= \{n \forall |x(n)| > v\} \end{aligned} \quad (4)$$

where $w(n)$ is noise component, $q(n)$ is quantization noise which is usually very small [5], $\varphi_1(v)$ and $\varphi_2(v)$ are the index sets of OFDM samples. It assumed that the receiver has the knowledge of the two sets. It is clear that due to the presence of the inflexion point v, small and large parts of the signal can be treated with different scales; enlarging small amplitudes by $1/u$ while compressing large amplitudes by u, which gives more flexibility and freedom in designing the companding form in order to meet the given system requirements such as PAPR reduction, signal average power, Power amplifier characteristics, and BER, and hence, leads to a better performance. However, taking into account the more accurate case that OFDM signal consists of three parts: small amplitudes, large amplitudes, and average amplitudes, more design flexibility and performance enhancement can be achieved if each one of these parts treated

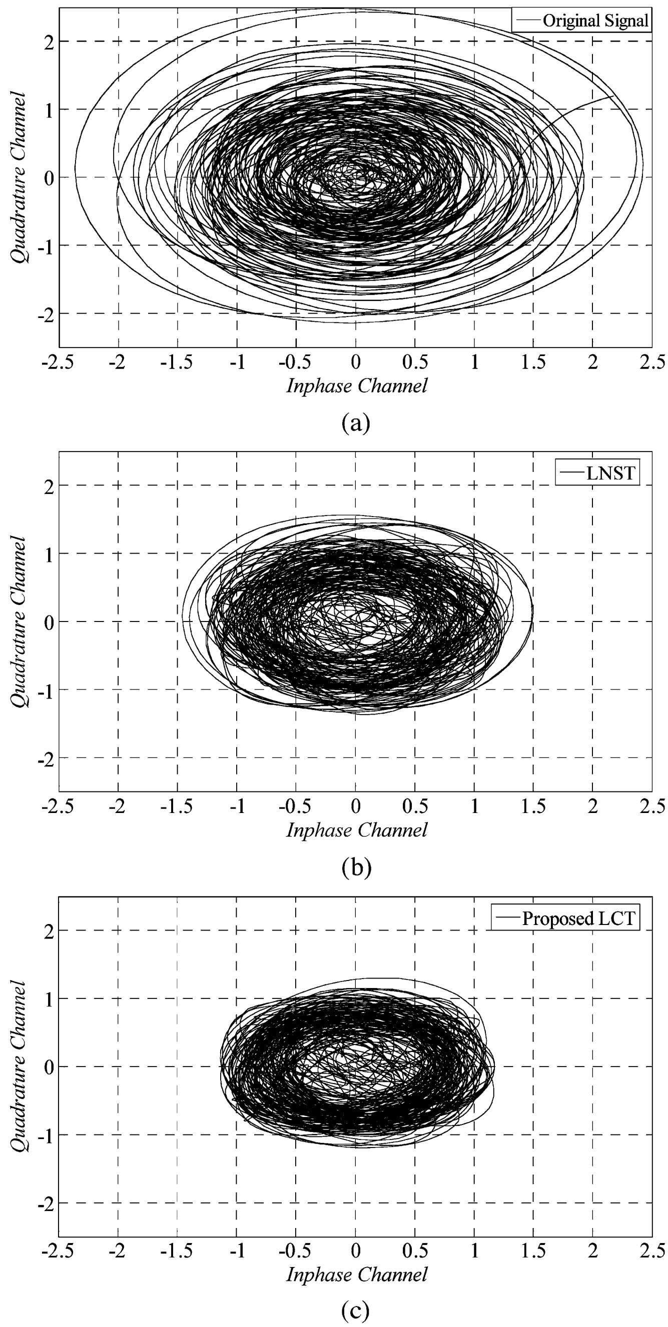

Fig. 3. (a) The original OFDM; (b) companded OFDM signal by LNST; (c) companded OFDM signal by the proposed transform.

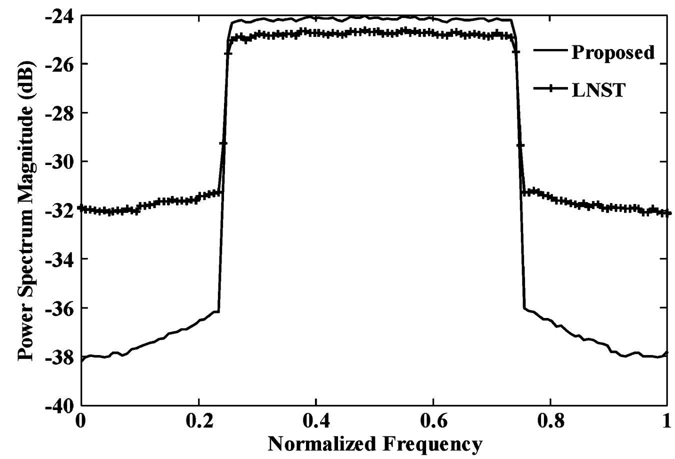

Fig. 4. Power spectrums of LNST and proposed transforms.

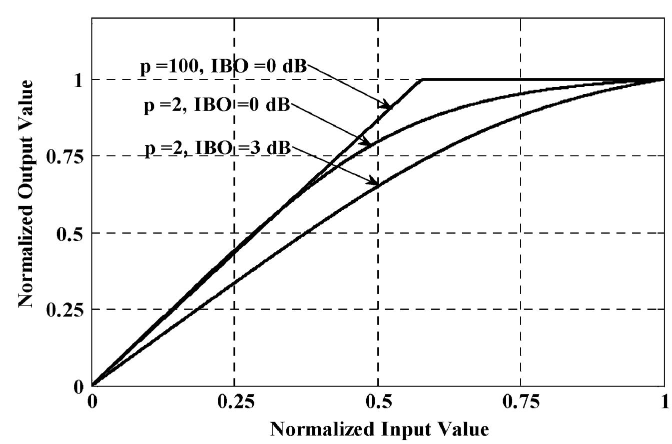

Fig. 5. SSPA characteristics.

independently with a different scale. To satisfy this, a new linear companding transform (LCT) with two inflexion points is proposed, the new transform is

$$y(n) = \begin{cases} u_1 \cdot x(n) & |x(n)| \le v_1 \\ u_2 \cdot x(n) & v_1 < |x(n)| \le v_2 \\ u_3 \cdot x(n) & |x(n)| > v_2 \end{cases} \tag{5}$$

$$\tilde{x}(n) = \begin{cases} \frac{1}{u_1} \cdot R(n) & n \in \varphi_1(v_1) \\ \frac{1}{u_2} \cdot R(n) & n \in \varphi_2(v_{1,2}) \\ \frac{1}{u_3} \cdot R(n) & n \in \varphi_3(v_2) \end{cases}$$

$$\begin{aligned} R(n) &= y(n) + w(n) + q(n) \\ \varphi_1(v_1) &= \{n \forall |x(n)| \le v_1\} \\ \varphi_2(v_{1,2}) &= \{v_1 < n \forall |x(n)| \le v_2\} \\ \varphi_3(v_2) &= \{n \forall |x(n)| > v_2\} \end{aligned} \tag{6}$$

where $u_1 > 1$ and $u_3 < 1$. Regarding u_2, setting its value to unity can effectively reduce the undesired effect of noise transformation at the receiver since average amplitudes are scaled with unity and hence, no inverse scaling is required at the receiver. Fig. 2 shows profiles of both transforms where $A = \max\{|x(n)|\}$, it is clear that with two inflexion points, more design flexibility is available and hence a better tradeoff between PAPR and BER can be achieved. Fig. 3 shows the original and companded OFDM signals on the complex plane, were transforms are designed to preserve the average power of input signal for case study, for practical purpose, the average power of companded signal should be selected to best fit for specific PA characteristics included in the system [12]. The variance of transformed noise at the receiver along with the average power of companded signal, are derived in the Appendix A. It is obvious that the companded signal by proposed LCT has the lowest PAPR "smallest radius," since LCT allows for more reduction of

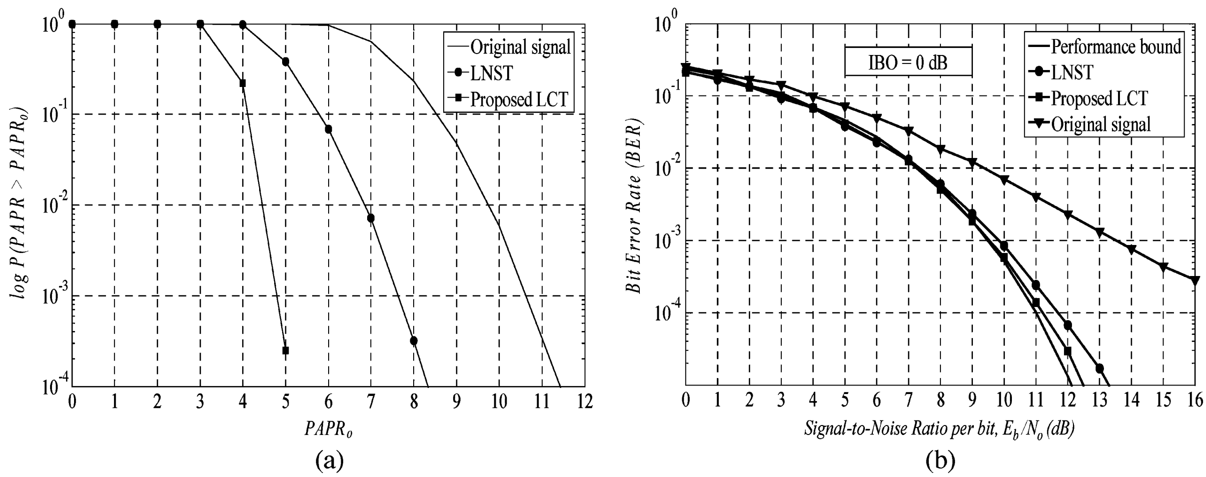

Fig. 6. Performance of LNST and proposed LCT. (a) Simulated CCDFs of LNST and proposed transform. (b) BER performance in nonlinear AWGN channel.

PAPR by extra compression of large amplitudes and by extra enlargement of small amplitudes without affecting average amplitudes and hence, reallocate power among all subcarriers. Moreover, the flexibility of the proposed transform allows reducing abrupt jumps in the transformed signal, which leads to better power spectrum as depicted in Fig. 4. Since the receiver must have the knowledge of index sets, side information should be transmitted along with the signal. For LNST either $\varphi_1(v)$ or $\varphi_2(v)$ can be transmitted as side information on dedicated subcarriers or imbedded in training sequences [12]. Specifically if v is set to be equal to the square root of signal average power, then transmitting $\varphi_2(v)$ will result in less overhead to be transmitted since it contains a smaller number of indices. This is because the samples of large amplitudes are usually occurring with low probability. Regarding the proposed transform, advantages of the extra inflexion point come at the price of another index set that should be transmitted.

IV. Nonlinear Power Amplifier

A widely accepted memoryless solid-state power amplifier (SSPA) model [19], which is extensively used in investigating PAPR of OFDM signals, is Rapp model [20], where a memoryless nonlinearity is assumed. Therefore, the PA has a frequency-nonselective response. Representing the complex envelope of the input signal into the amplifier as

$$y(t) = |y(t)|\, e^{j\phi(t)} \tag{7}$$

the transmitted output signal according to the model can be expressed as

$$y_{tx}(t) = \frac{a\,|y(t)|}{\left[1 + \left(\frac{|y(t)|}{A_{sat}}\right)^{2p}\right]^{\frac{1}{2p}}} e^{j\phi(t)} \tag{8}$$

where, a is the amplifier gain, A_{sat} is the saturation level, and p is a positive number to control nonlinearity characteristics of the amplifier. According to this model, SSPA introduces no phase distortion and only the AM/AM conversion is produced.

TABLE I
Transforms Parameters and PAPR Reduction

Transform	Parameters $A = \max\{\lvert x(n)\rvert\}$	PAPR reduction (average value)
LNST	$u = 0.625$ [12] $v = \sqrt{P_{in}}$	50%
Proposed LCT	$u_1 = 2$ $u_2 = 1$ $u_3 = 0.45$ v_1 =20% of A v_2 = 40.% of A	70%

Input power Backoff (IBO) can be expressed as

$$IBO = \frac{A_{sat}^2}{P_{in}} \tag{9}$$

where P_{in} is the average power of the input signal. According to (9), the average power of the input signal should be scaled with the proper value for a given PA characteristics (A_{sat}, IBO) [21]. Fig. 5 shows the characteristics of SSPA model, as it is shown, for large values of p, the model converges to a hard limiting amplifier that is exactly linear until it reaches its output saturation level. A good approximation of existing amplifiers is obtained by choosing p in the range of 2 to 3 [22]. In this paper, we chose $p = 2$.

V. Performance Evaluation

In order to evaluate and compare the performance of the proposed transform and examine its impact on the system, a MATLAB simulation was performed, assuming nonlinear AWGN channel and using randomly generated data bits with DQPSK modulation. Symbols are transmitted over 64 subcarriers with 256-point IFFT/FFT (oversampling factor equal to 4). The LNST & proposed LCT Transforms parameters and achieved PAPR reduction are tabulated in Table I. Simulation results are presented in Fig. 6.

As it is shown in Table I, the PAPR reduction capability "power efficiency" of the proposed transform reaches 70% of original PAPR on the average, which is 20% more than LNST capability. This is demonstrated in Fig. 6(a), which shows simulated Complementary Cumulative Distribution Function (CCDF) of each CT. CCDFs where obtained by randomly generating 100,000 OFDM symbols at each $PAPR_o$ value and counting number of blocks that exceed this value.

Regarding BER efficiency which depicted in Fig. 6(b), the proposed transform has an excellent performance. Specifically, for a target BER of 10^{-5} with IBO of 0 dB, the proposed transform requires a signal-to-noise ratio (SNR) of 12.5 dB which is just 0.38 dB above performance bound obtained by disabling the SSPA and transmitting original OFDM signal directly. Compared to the proposed LCT, the LNST requires 13.29 dB which is 1.17 dB above the bound.

VI. Conclusion

In this paper, a new linear companding transform is proposed with two inflexion points in order to increase the flexibility of companding design, results show that the proposed transform has a higher PAPR reduction capability and better BER performance than LNST, with less spectral broadening. In general, with the aid of two inflexion points, different signal levels can be scaled independently of each other. Thus, the proposed transform can be designed to meet system requirements, power amplifier characteristics, and achieve an excellent tradeoff between PAPR reduction and BER performance. Furthermore, the proposed transform is simple to implement and has no limitations on the system parameters such as number of subcarriers modulation order, or constellation type.

Appendix A

The transform gain G is defined as

$$G = \frac{PAPR(x(n))}{PAPR(y(n))} \tag{A.1}$$

thus, $G = (A^2/A'^2) \cdot (M'^2/M^2)$, where $M^2 = P_{in}$ and M'^2, denote the average power of the original and companded signals, respectively.

$$A'^2 = \max\left\{(u_1v_1)^2, (u_2v_2)^2, (u_3A)^2\right\} \tag{A.2}$$

is the peak power of companded signal, and A^2 is the peak power of original signal.

With the probability distribution function of OFDM signal denoted as $f(x)$, which has Gaussian distribution function

$$f(x) = \frac{1}{\sqrt{2\pi}M^2} e^{-\frac{x^2}{2M^2}}, \tag{A.3}$$

the average power of the companded signal can be written as

$$\begin{aligned} M'^2 =& 2\int_0^{v_1} u_1^2 x^2 f(x)dx + 2\int_{v_1}^{v_2} u_2^2 x^2 f(x)dx \\ &+ 2\int_{v_2}^{A} u_3^2 x^2 f(x)dx \\ =& 2M^2u_1^2\left[\frac{1}{2} - Q\left(\frac{v_1}{M}\right) - \frac{v_1}{\sqrt{2\pi}M}e^{-\frac{v_1^2}{2M^2}}\right] \\ &+ 2M^2u_2^2\left[Q\left(\frac{v_1}{M}\right) - Q\left(\frac{v_2}{M}\right)\right. \\ &\left. - \frac{1}{\sqrt{2\pi}M}\left(v_2e^{-\frac{v_2^2}{2M^2}} - v_1e^{-\frac{v_1^2}{2M^2}}\right)\right] \\ &+ 2M^2u_3^2\left[Q\left(\frac{v_2}{M}\right) - Q\left(\frac{A}{M}\right)\right. \\ &\left. - \frac{1}{\sqrt{2\pi}M}\left(Ae^{-\frac{A^2}{2M^2}} - v_2e^{-\frac{v_2^2}{2M^2}}\right)\right]. \end{aligned} \tag{A.4}$$

The variance of the transformed noise term at the receiver can be written as

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2, \tag{A.5}$$

where

$$\begin{aligned} \sigma_1^2 &= E\left\{(1/u_1^2)\,|\,|x(n)| \le v_1\right\}\left(\sigma_o^2 + \frac{A'^2}{A^2}\sigma_q^2\right) \\ &= (2/u_1^2)\left[\frac{1}{2} - Q\left(\frac{v_1}{M}\right)\right]\left(\sigma_o^2 + \frac{A'^2}{A^2}\sigma_q^2\right) \\ \sigma_2^2 &= E\left\{(1/u_2^2)\,|v_1 < |x(n)| \le v_2\right\}\left(\sigma_o^2 + \frac{A'^2}{A^2}\sigma_q^2\right) \\ &= (2/u_2^2)\left[Q\left(\frac{v_1}{M}\right) - Q\left(\frac{v_2}{M}\right)\right]\left(\sigma_o^2 + \frac{A'^2}{A^2}\sigma_q^2\right) \\ \sigma_3^2 &= E\left\{(1/u_3^2)\,|\,|x(n)| > v_2\right\}\left(\sigma_o^2 + \frac{A'^2}{A^2}\sigma_q^2\right) \\ &= (2/u_3^2)\left[Q\left(\frac{v_2}{M}\right) - Q\left(\frac{A}{M}\right)\right]\left(\sigma_o^2 + \frac{A'^2}{A^2}\sigma_q^2\right) \end{aligned}$$

and σ_o^2 and σ_q^2, are variances of the Gaussian noise and the quantization error, respectively.

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