PAPR Analysis of DHT-Precoded OFDM System for M-QAM

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Abstract—**High Peak to Average Power Ratio (PAPR) is one of the major drawbacks in Orthogonal Frequency Division Multiplexing (OFDM). The High PAPR increases the complexity of Analogue to Digital (A/D) and Digital to Analogue (D/A) convertors and also reduces the efficiency of RF High Power Amplifier (HPA). In this paper we present an analysis of Discrete Hartley Transform (DHT) precoded OFDM system using M-QAM (where M=16, 32, 64, 256). We compare the computer simulation results of DHT precoded OFDM system with DFT precoded OFDM system, Walsh Hadamard Transform (WHT) precoded OFDM system, Selected Mapping (SLM) based OFDM system and OFDM conventional. Simulation results show that the PAPR of DHT precoded OFDM system is lower than WHT precoded OFDM system, SLM-OFDM system and OFDM conventional. We also concluded in this paper that DFT precoded OFDM system has zero PAPR.**

Index Terms-- **PAPR, DFT precoder, DHT precoder, HPA**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier transmission scheme that has become the technology of choice for next generation wireless and wireline digital communication systems because of its high speed data rates, high spectral efficiency, high quality service and robustness against narrow band interference and frequency selective fading [1].

OFDM thwarts Inter Symbol Interference (ISI) by inserting a Guard Interval (GI) using a Cyclic Prefix (CP) and moderates the frequency selectivity of the Multi Path (MP) channel with a simple equalizer. This leads to cheap hardware implementation and makes simpler the design of the receiver. OFDM is widely adopted in various communication standards like Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), Digital Subscriber Lines (xDSL), Wireless Local Area Networks (WLAN), Wireless Metropolitan Area Networks (WMAN), Wireless Personal Area Networks (WPAN) and even in the beyond 3G Wide Area Networks (WAN) etc. Additionally, OFDM is a strong candidate for Wireless Asynchronous Transfer Mode (WATM).

However, among others, the Peak to Average Power Ratio (PAPR) is still one of the major drawbacks in the transmitted OFDM signal [2]. Therefore, for zero distortion of the OFDM signal, the HPA must not only operate in its linear region but also with sufficient back-off. Thus, the RF High Power Amplifier (HPA) with a large dynamic range are required for OFDM system. These amplifiers are very expensive and are major cost component of the OFDM system. Thus, if we reduce the PAPR it not only means that we are reducing the cost of OFDM system and reducing the complexity of the Analogue to Digital (A/D) and Digital to Analogue (D/A) convertors, but also increasing the transmit power, thus, for same range improving received Signal to Noise Ratio (SNR), or for the same SNR improving range.

A large number of PAPR reduction techniques have been proposed in the literature. Among them, schemes like constellation shaping [3], coding schemes [4,5], phase optimization [6], nonlinear companding transforms [7], Tone Reservation (TR) and Tone Injection (TI) [8,9], clipping and filtering [10], Partial Transmit Sequence (PTS) [11], Precoding based Selected Mapping (PSLM) [12], precoding based techniques [13] and Selected Mapping (SLM) [14] are popular.

In [10] Wang and Tellambura proposed a soft clipping technique which preserves the phase and clips only the amplitude. They also put a lot of effort to characterize the performance and discover some properties to simplify the job. However, the PAPR gain is only estimated by simulations and is limited to a specific class of modulation technique.

In [11] Han and Lee proposed a PAPR reduction technique based on Partial Transmit Sequence technique in which they divide the frequency bins into sub blocks and then they multiply each sub-block with a constant phase shift. Choosing the appropriate phase shift values reduces PAPR. The most critical part of this technique is to find out the optimal phase value combination and in this regard they also proposed a simplified search method and evaluated the performance of the proposed technique.

In [12] authors developed PSLM technique for PAPR reduction. In this technique Zadoff-Chu based precoder is applied after the multiplication of phase rotation factor and before the IFFT in the SLM-OFDM system. The proposed PSLM technique is signal independent and it does not require any complex optimization technique. In [13] authors proposed Zero PAPR Zadoff-Chu precoder based technique for Single Carrier Frequency Division Multiple Access (SC-FDMA). This technique is efficient, signal independent, distortionless, it does not require any optimization algorithm and PAPR is completely eliminated.

This paper analyses PAPR of the DHT-Precoding based OFDM system for PAPR reduction. It is organized as follows: Section II describes the basics of the OFDM system

and PAPR reduction, In Section III we present the proposed system model for PAPR reduction, and Section IV presents computer simulation results and section V concludes the paper.

II. OFDM SYSTEM & PAPR REDUCTION

The OFDM system splits the high speed data stream into a number of parallel low data rate streams and these low rates data streams are transmitted simultaneously over a number of orthogonal subcarriers.
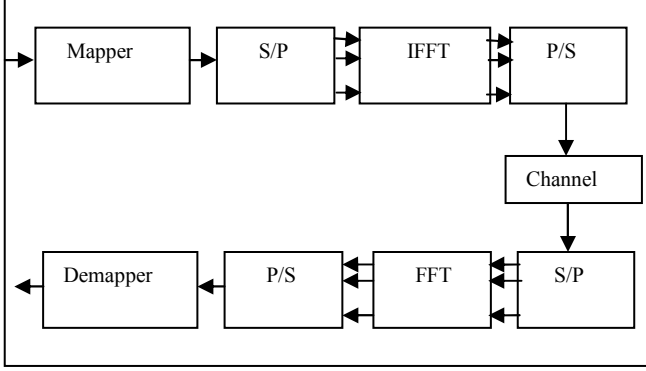

Fig. 1.Block diagram of OFDM system

Fig. 1 illustrates the block diagram of an OFDM system. Baseband modulated symbols are passed through serial to parallel converter which generates complex vector of size N. We can write the complex vector of size N as $X = [X_0, X_1, X_2... X_{N-1}]^T$. X is then passed through the IFFT block. The complex baseband OFDM signal with N subcarriers can be written as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k . e^{j2\pi\frac{n}{N}k} , n=0, 1, 2... N-1 \qquad (1)$$

Here $j=\sqrt{-1}$ and the PAPR of OFDM signal in (1) can be written as

$$PAPR = \frac{max|x_n|^2}{E[|x_n|^2]} \qquad (2)$$

where $E[.]$ denotes expectation and the Complementary Cumulative Distribution Function (CCDF) for an OFDM signal can be written as

$$P(PAPR > PAPR_0) = 1 - (1 - e^{-PAPR_0})^N \qquad (3)$$

where $PAPR_0$ is the clipping level. This equation can be read as the probability that the PAPR of a symbol block exceeds some clip level $PAPR_0$.

III. PROPOSED MODEL

A. The Precoding Based OFDM system

Fig. 2 shows the block diagram of Precoding Based OFDM System. We implemented the Precoding matrix P of dimension $N \times N$ before the IFFT to reduce the PAPR.
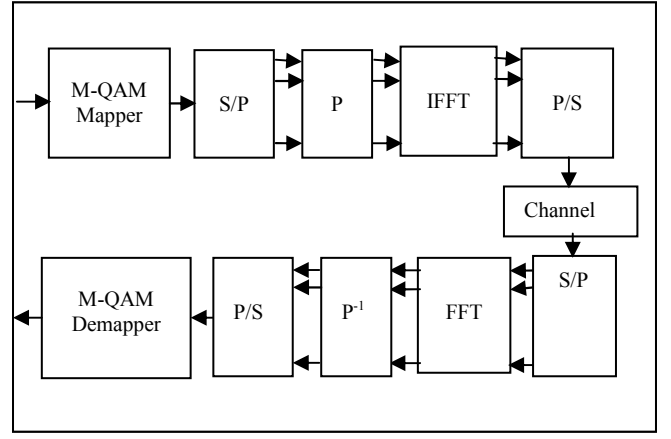

Fig. 2.Block diagram of Pre IFFT based OFDM system

The Precoding matrix P can be written as

$$P = \begin{bmatrix} p_{00} & p_{01} & ... & p_{0(N-1)} \\ p_{10} & p_{11} & ... & p_{1(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ p_{(N-1)0} & p_{(N-1)1} & ... & p_{(N-1)(N-1)} \end{bmatrix} \qquad (4)$$

where P is a Precoding Matrix of size $N \times N$ is shown in equation (4). The complex baseband OFDM signal with N subcarriers can be written as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} P.X_k . e^{j2\pi k\Delta ft}, \quad 0 \leq t \leq NT \qquad (5)$$

We can express modulated OFDM vector signal with N subcarriers as.

$$x_N = IFFT\{P.X_N\} \qquad (6)$$

The PAPR of OFDM signal in (5) can be written as

$$PAPR = \frac{max|x(t)|^2}{E[|x(t)|^2]} \qquad (7)$$

B. The Discrete Fourier Transform (DFT) Precoding

The DFT of a sequence of length N can be defined as

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk} , k=0, 1 ... N-1 \qquad (8)$$

and IDFT can be written as

$$x(n) = \frac{1}{N}\sum_{n=0}^{N-1} X(k)e^{j2\pi nk} , k=0, 1 ... N-1 \qquad (9)$$

where $p_{mn} = e^{\frac{-j2\pi mn}{N}}$, m and n are integers from 0 to $N-1$ and P is precoding matrix of size $N \times N$ shown in equation(4).

C. The Discrete Hartley Transform (DHT) Precoding

The DHT is a linear transform. In DHT N real numbers $x_0, x_1, ..., x_{N-1}$ are transformed in to N real numbers $H_0, H_1, ..., H_{N-1}$. According to [15] the N-point DHT can be defined as

$$H_k = \sum_{n=0}^{N-1} x_n \left[cos\left(\frac{2\pi nk}{N}\right) + sin\left(\frac{2\pi nk}{N}\right) \right]$$

$$= \sum_{n=0}^{N-1} x(n) . \ cas\left(\frac{2\pi nk}{N}\right) \qquad (10)$$

Where $cas\ \theta = cos\ \theta + \ sin\ \theta$ and k = 0, 1… N-1

$$p_{m,n} = cas\left(\frac{2\pi mn}{N}\right) \qquad (11)$$

P is precoding matrix of size $N×N$ shown in equation (4), m and n are integers from 0 to N-1. The DHT is also invertible transform which allows us to recover the x_n from H_k and inverse can be obtained by simply multiplying DHT of H_k by $\frac{1}{N}$.

IV. SIMULATION RESULTS

We performed extensive simulations in MATLAB® in order to evaluate the performance of DHT-Precoder based OFDM system. To show the PAPR analysis of DHT-Precoded OFDM system, data is generated randomly then modulated by M-QAM (where M=16, 32, 64, 256). We compared our simulation results with DFT-Precoded OFDM system on one hand and also we compared our simulation results with WHT-Precoder OFDM system, SLM-OFDM system and OFDM-Original system. To show the overall performance of the DHT-Precoder based OFDM system for PAPR reduction in MATLAB® we considered M-QAM for N=64. It is to be noted that M-QAM has itself PAPR. Table 1 summarises the PAPR of M-QAM (M=16, 32, 64, 256).

TABLE 1
PAPR FOR M-QAM

Type Of Modulation(M-QAM)	PAPR(dB)
4-QAM	zero
16-QAM	2.55
32-QAM	3.30
64-QAM	3.68
256-QAM	4.23

Figure.3 shows the CCDF comparisons of DHT-Precoder Based OFDM system with DFT-Precoder Based OFDM system, WHT-Precoder Based OFDM system, SLM-OFDM (with V=2) system and OFDM-Original system for N=64. At clip rate of 10^{-2}, the PAPR gain of 3dB, 2.5dB and 2dB is achieved when we compare DHT-Precoder Based OFDM system with OFDM-Original system, WHT-Precoder Based OFDM system and SLM-OFDM (with V=2) system respectively for 16-QAM. However, DHT-Precoding based OFDM system does not perform as well as DFT-Precoding based OFDM system, which is better than DHT-Precoding based OFDM system by 3dB at same clip rate. Seen in comparison to table 1, DFT-Precoding based OFDM system given rise to nearly same, where theoretical PAPR of 16-QAM is 2.55dB.

Figure.4 shows the CCDF comparisons of DHT-Precoder Based OFDM system with DFT-Precoder Based OFDM system, WHT-Precoder Based OFDM system, SLM-OFDM (with V=2) system and OFDM-Original system for N=64.
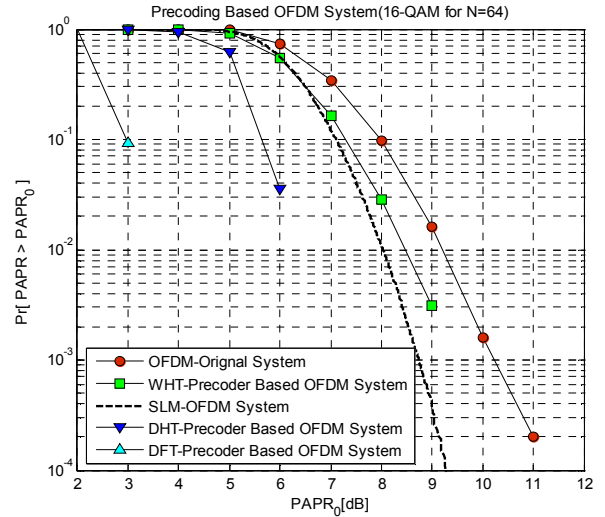

Fig.3. CCDF comparison of DHT-Precoder Based OFDM System with DFT-Precoder Based OFDM system, WHT-Precoder Based OFDM System, SLM-OFDM (V=2) System and OFDM Original System for 16-QAM
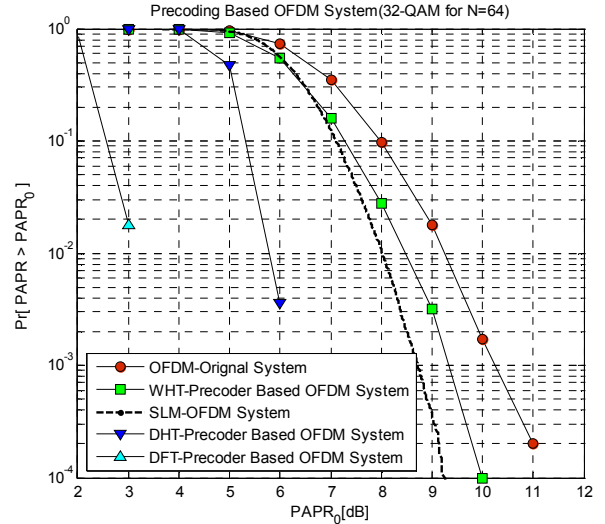

Fig.4. CCDF comparison of DHT-Precoder Based OFDM System with DFT-Precoder Based OFDM system, WHT-Precoder Based OFDM System, SLM-OFDM (V=2) System and OFDM Original System for 32-QAM
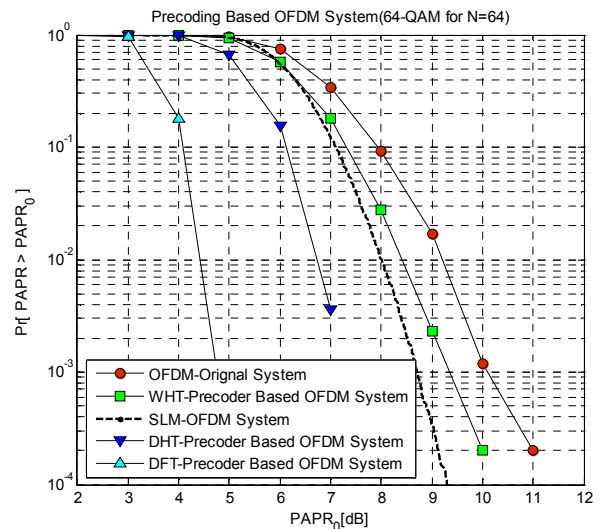

Fig.5. CCDF comparison of DHT-Precoder Based OFDM System with DFT-Precoder Based OFDM system, WHT-Precoder Based OFDM System, SLM-OFDM (V=2) System and OFDM Original System for 64-QAM
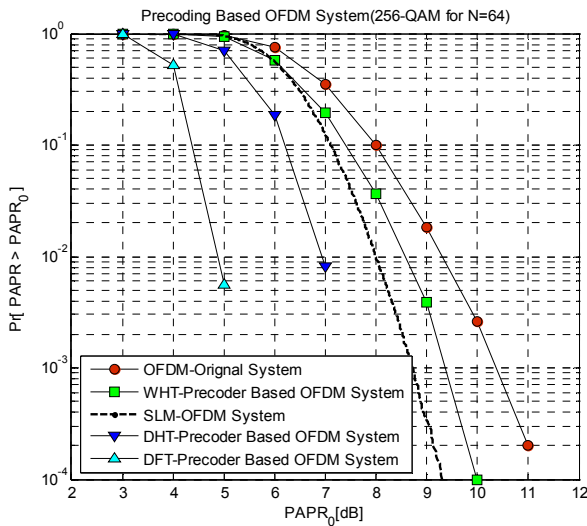
Fig.6. CCDF comparison of DHT-Precoder Based OFDM System with DFT-Precoder Based OFDM system, WHT-Precoder Based OFDM System, SLM-OFDM (V=2) System and OFDM Original System for 256-QAM

At clip rate of 10^{-2}, the PAPR gain of 3.5dB, 2.7dB and 2.3dB is achieved when we compare DHT-Precoder Based OFDM system with OFDM-Original system, WHT-Precoder Based OFDM system and SLM-OFDM (with V=2) system respectively for 32-QAM. However, DHT-Precoding based OFDM system does not perform as well as DFT-Precoding based OFDM system, which is better than DHT-Precoding based OFDM system by 2.7dB at same clip rate. Seen in comparison to table 1, DFT-Precoding based OFDM system given rise to nearly same, where theoretical PAPR of 32-QAM is 2.30dB.

Figure.5 shows the CCDF comparisons of DHT-Precoder Based OFDM system with DFT-Precoder Based OFDM system, WHT-Precoder Based OFDM system, SLM-OFDM (with V=2) system and OFDM-Original system for N=64. At clip rate of 10^{-3}, the PAPR gain of 3dB, 3.2dB and 1.1dB is achieved when we compare DHT-Precoder Based OFDM system with OFDM-Original system, WHT-Precoder Based OFDM system and SLM-OFDM (with V=2) system respectively for 64-QAM. However, DHT-Precoding based OFDM system does not perform as well as DFT-Precoding based OFDM system, which is better than DHT-Precoding based OFDM system by 2dB at same clip rate. Seen in comparison to table 1, DFT-Precoding based OFDM system given rise to nearly same, where theoretical PAPR of 64-QAM is 3.68dB.

Figure.6 shows the CCDF comparisons of DHT-Precoder Based OFDM system with DFT-Precoder Based OFDM system, WHT-Precoder Based OFDM system, SLM-OFDM (with V=2) system and OFDM-Original system for N=64. At clip rate of 10^{-2}, the PAPR gain of 2.4dB, 1.7dB and 1.8dB is achieved when we compare DHT-Precoder Based OFDM system with OFDM-Original system, WHT-Precoder Based OFDM system and SLM-OFDM (with V=2) system respectively for 256-QAM. However, DHT-Precoding based OFDM system does not perform as well as DFT-Precoding based OFDM system, which is better than DHT-Precoding based OFDM system by 2dB at same clip rate. If comparison to table 1, DFT-Precoding based OFDM system given rise to

nearly same, theoretical PAPR of 256-QAM is 4.23dB. Thus, it can be concluded that DHT-Precoding based OFDM system performs better than most of the popular PAPR reduction schemes for OFDM system, except DFT-Precoding based OFDM system.

V. CONCLUSION

In this paper, we analysed the PAPR of DHT-Precoded OFDM system for M-QAM (where M=16, 32, 64, 256). Matlab simulation shows that DHT-Precoded OFDM System shows better PAPR gain as compared to OFDM-Original system, WHT-Precoder Based OFDM system and SLM-OFDM (with V=2) system respectively. Thus, it is concluded that DHT Precoder Based OFDM System shows better PAPR reduction then WHT-Precoder Based OFDM System, SLM-OFDM System and OFDM-Original system for M-QAM. Additionally, the DHT-Precoded OFDM system does not require any power increase, complex optimization and side information to be sent for the receiver.

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