# Stationary and Non-Stationary noise removal from Cardiac Signals using a Constrained Stability Least Mean Square Algorithm

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*Abstract*—**Adaptive filter is a primary method to filter ECG signal, because it does not need the signal statistical characteristics. In this paper we present a novel adaptive filter for removing the artifacts from ECG signals based on Constrained Stability Least Mean Square (CSLMS) algorithm. This algorithm is derived based on the minimization of the squared Euclidean norm of the difference weight vector under a stability constraint defined over the posteriori estimation error. The adaptive filter essentially minimizes the mean-squared error between a primary input, which is the noisy ECG, and a reference input, which is either noise that is correlated in some way with the noise in the primary input or a signal that is correlated only with ECG in the primary input. Different filter structures are presented to eliminate the diverse forms of noise. Finally, we have applied this algorithm on ECG signals from the MIT-BIH data base and compared its performance with the conventional LMS algorithm. The results show that the performance of the CSLMS based algorithm is superior to that of the LMS based algorithm in noise reduction.**

**Keywords-adaptive noise cancelation, artifacts, ECG signals, LMS algorithm, Noise cancelation.**

## I. Introduction

Baseline wander and powerline interference reduction is the first step in all electrocardiographic (ECG) signal processing. The baseline wander is caused by varying electrode-skin impedance, patients movements and breath. This kind of disturbances is especially present in exercise electrocardiography, as well as during ambulatory and Holter monitoring. The ECG signal is also degraded by additive 50 or 60 Hz powerline (AC) interference. This kind of disturbance can be modeled by a sinusoid with respective frequency and random phase. These two artifacts are the dominant artifacts and strongly affects the ST segment, degrades the signal quality, frequency resolution, produces large amplitude signals in ECG that can resemble PQRST waveforms and masks tiny features that are important for clinical monitoring and diagnosis. Cancelation of these artifacts in ECG signals is an important task for better diagnosis. Hence the extraction of high-resolution ECG signals from recordings which are contaminated with background noise is an important issue to investigate. The goal of ECG signal enhancement is to separate the valid signal components from the undesired artifacts, so as to present an ECG that facilitates easy and accurate interpretation. Many approaches have been reported in the literature to address ECG enhancement using both adaptive and non-adaptive techniques [1]-[10], adaptive filtering techniques permit to the detect time varying potentials and to track the dynamic variations of the signals. In [2], Thakor et al. proposed an LMS based adaptive recurrent filter to acquire the impulse response of normal QRS complexes and then applied it for arrhythmia detection in ambulatory ECG recordings.

Apart from these several adaptive signal processing techniques are also published, e.g., NLMS algorithm with decreasing step size, which converge to the global minimum [11], a variable step size NLMS algorithm with faster convergence rate [12], Costa et al. in [13] proposed a noise resilient variable step size LMS which is specially indicated for biomedical applications. Also several modifications are presented in literature to improve the performance of the LMS algorithm [14]-[20]. Recently in [21] Rahman et al. presented several less computational complex adaptive algorithms in time domain, but these algorithms exhibits slower convergence rate.

This paper presents a novel adaptation for filtering Cardiac signals in continuous stationary and non-stationary environments in biotelemetry systems, which are characterized by sudden changes of the signal statistics due to physiological, non-physiological reasons and noises due to free space propagation. The considered CSLMS algorithm is based on the concept of difference quantities and the constraint of equilibrium in the sequence of a posteriori estimation errors [22]. The method, which applies nonlinearities to the error and input signal sequences, which can be derived using the Lagrange multiplier method as a generalization of the normalized LMS (NLMS). Under certain conditions the adaptive noise cancelers (ANC) based on the CSLMS algorithm shows improved performance by decreasing the excess mean squared error and misadjustment compared to conventional algorithms like, LMS and NLMS algorithms. Thus far, to the best of the authors knowledge, CSLMS algorithm is not used in the contest of ECG signal noise cancelation. In this paper various adaptive filter structures are presented to eliminate different kinds of noises from cardiac signals. Finally to study the performance of the filter structures which effectively remove the artifacts from the ECG signal we carried out simulations on MIT-BIH database. The simulation results shows that the performance of CSLMS based algorithms is better than the LMS counterpart.

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## II. Proposed Implementation

Consider a length $L$, LMS based adaptive filter, depicted in Fig. 1, that takes an input sequence $x(n)$ and updates the weights as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu\,\mathbf{x}(n)\,e(n), \tag{1}$$

where, $\mathbf{w}(n) = [w_0(n)\ w_1(n) \cdots w_{L-1}(n)]^t$ is the tap weight vector at the $n$th index, $\mathbf{x}(n) = [x(n)\ x(n-1) \cdots x(n-L+1)]^t$, error signal $e(n) = d(n) - \mathbf{w}^t(n)\,\mathbf{x}(n)$, with $d(n)$ being so-called the desired response available during initial training period and $\mu$ denoting so-called step-size parameter.

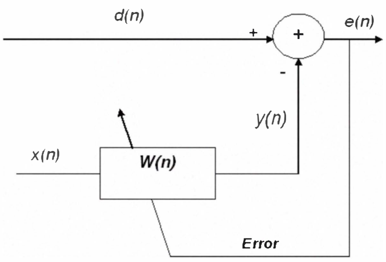


Fig. 1. Adaptive filter structure

In order to remove the noise from the ECG signal, the ECG signal $s_1(n)$ corrupted with noise signal $p_1(n)$ is applied as the desired response $d(n)$ to the adaptive filter shown in Fig. 1. If the noise signal $p_2(n)$, possibly recorded from another generator of noise that is correlated in some way with $p_1(n)$ is applied at the input of the filter, i.e., $x(n) = p_2(n)$ the filter error becomes $e(n) = [s_1(n) + p_1(n)] - y(n)$. Where, $y(n)$ is the filter output and it is given by,

$$y(n) = \mathbf{w}^t(n)\mathbf{x}(n), \tag{2}$$

Since the signal and noise are uncorrelated, the mean-squared error (MSE) becomes

$$E[e^2(n)] = E\{[s_1(n) - y(n)]^2\} + E[p_1^2(n)] \tag{3}$$

Minimizing the MSE results in a filter output which is the best least-squares estimate of the signal $s_1(n)$.

Normalized LMS (NLMS) algorithm is another class of adaptive algorithm used to train the coefficients of the adaptive filter. This algorithm takes into account variation in the signal level at the filter output and selecting the normalized step size parameter that results in a stable as well as fast converging algorithm. The weight update relation for NLMS algorithm is as follows

$$\mathbf{w}(n+1) = \mathbf{w}(n) + [\frac{\mu}{p + \mathbf{x}^t(n)\,\mathbf{x}(n)}]\,\mathbf{x}(n)\,e(n), \tag{4}$$

The variable step can be written as,

$$\mu(n) = \frac{\mu}{p + \mathbf{x}^t(n)\,\mathbf{x}(n)} \tag{5}$$

Here $\mu$ is fixed convergence factor to control maladjustment.

A common major drawback of adaptive noise canceler based on LMS and NLMS algorithms is the large value of excess mean-square error which results in signal distortion in the noise-canceled signal. In the CSLMS algorithm the time-varying step-size that is inversely proportional to the squared norm of the difference between two consecutive input vectors rather than the input data vector as in the NLMS. This algorithm provides significant improvements in decreasing mean-squared error (EMSE) and consequently minimizing signal distortion [22].

The weight update relation for CSLMS algorithm is as follows,

$$\mathbf{w}(n+1) = \mathbf{w}(n) + [\frac{\delta\mathbf{x}(n)\,\delta\mathbf{e}(n)}{\|\delta\mathbf{x}(n)\|^2}] \tag{6}$$

Where $\delta x(n) = x(n) - x(n-1)$ is the difference between two consecutive input vectors. Also $\delta e(n) = e(n) - e(n-1)$ is the difference in the priori error sequence.
The weight adaptation rule can be made more robust by introducing a small $p$ and by multiplying the weight increment by a constant step size $\mu$ to control the speed of the adaptation. This gives the weight update relation for CSLMS algorithm in its final form as follows,

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu[\frac{\delta\mathbf{x}(n)\,\delta\mathbf{e}(n)}{p + \|\delta\mathbf{x}(n)\|^2}] \tag{7}$$

The parameter $p$ is set to avoid denominator being too small, step size parameter too big and to prevent numerical instabilities in case of a vanishingly small squared norm. The convergence characteristics of both the algorithms are shown in Fig. 2.

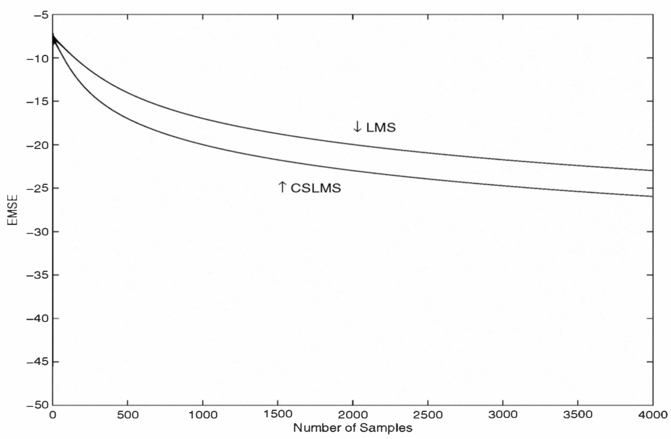


Fig. 2. Typical convergence curves of LMS and CSLMS for PLI cancellation

## III. Simulation Results

To show that CSLMS algorithm is really effective in clinical situations, the method has been validated using several ECG recordings with a wide variety of wave morphologies from MIT-BIH arrhythmia database. We used the benchmark MIT-BIH arrhythmia database ECG recordings as the reference for our work and real noise is obtained from MIT-BIH Normal Sinus Rhythm Database (NSTDB). In our simulations we consider both stationary (PLI) and non-stationary (BW) noises. The arrhythmia data base consists of 48 half hour excerpts of two channel ambulatory ECG recordings, which were obtained from 47 subjects, including 25 men aged 32-89 years, and women aged 23-89 years. The recordings were digitized at 360 samples per second per channel with 11-bit resolution over a 10 mV range. In our experiments we used a data set of five records (records 101, 102, 103, 104 and 105) but due to space constraint simulation results for record 105 are shown in this paper. In our simulation we collected 4000 samples of ECG signal, a random noise with variance ($\sigma$) of 0.001, 0.01 and 0.1 is added to the ECG signals to evaluate the performance of the algorithm in terms of minimum MSE (MMSE), MSE, excess MSE (EMSE) and misadjustment (M). For evaluating the performance of the proposed adaptive filter we have also measured the SNR improvement and compared with LMS algorithm. For all the figures $number\ of\ samples$ are taken on x-axis and $amplitude$ on y-axis, unless stated. Table I shows the comparison of MMSE, MSE, EMSE and M for LMS, NLMS and CSLMS algorithms. Table II gives the contrast of the considered algorithms in terms of SNR improvement (SNRI).

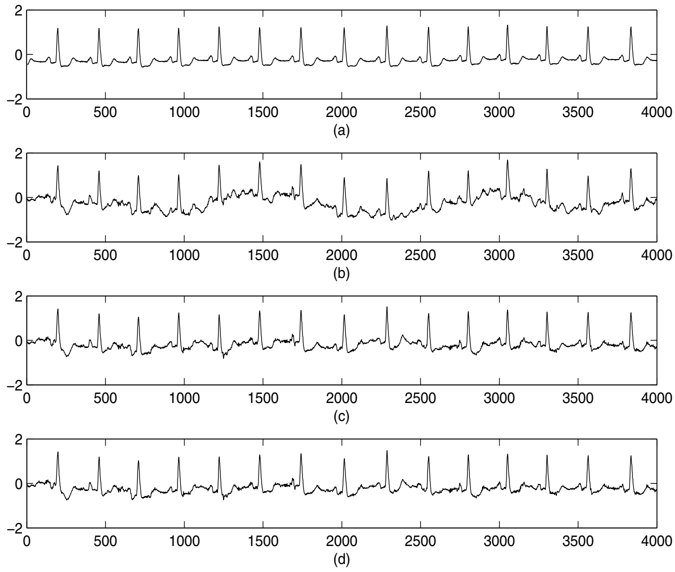

Fig. 3. Typical filtering results of baseline wander reduction (a)clean MIT-BIH record 105, (b) ECG with real BW, (c) recovered signal using LMS algorithm, (d) recovered signal using CSLMS algorithm.

### A. Baseline Wander Reduction

In this experiment, first we collected 4000 samples of the pure ECG signal from the MIT-BIH arrhythmia database(data105) and it is corrupted with real baseline wander (BW) taken from the MIT-BIH Noise Stress Test Database (NSTDB). This database was recorded at a sampling rate of 128Hz from 18 subjects with no significant arrhythmias. The contaminated ECG signal is applied as primary input to the adaptive filter of Fig.1. The real BW is given as reference signal. Different filter structures were implemented using the LMS and CSLMS algorithms to study the relative performance and results are plotted in Fig.3. On average LMS algorithm gets SNR improvement 3.1428dB, where as CSLMS gets 4.7613dB.

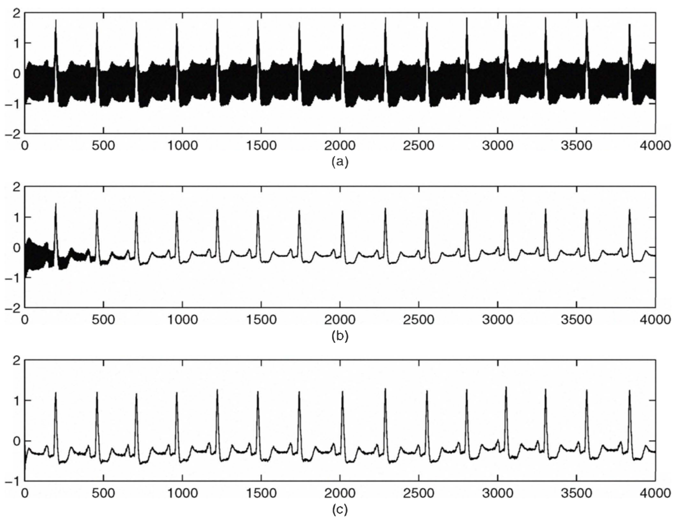

Fig. 4. Typical filtering results of PLI Cancelation (a)ECG with 60Hz noise, (b) recovered signal using LMS algorithm, (c) recovered signal using CSLMS algorithm.

### B. Adaptive Power-line Interference Canceler

To demonstrate power line interference (PLI) cancelation we have chosen MIT-BIH record number 105. The input to the filter is ECG signal corresponds to the data 105 corrupted with synthetic PLI with amplitude 1mv and frequency 60Hz, sampled at 200Hz. The reference signal is synthesized PLI, the output of the filter is recovered signal. These results are shown in Fig.4. In SNR measurements it is found that CSLMS algorithm gets SNR improvement 13.7365dB, where as the LMS algorithm improves 6.3702dB. Fig.5 shows the power spectrum of the noisy signal before and after filtering with LMS and CSLMS algorithms.

## IV. Conclusion

In this paper the process of noise removal from ECG signal using CSLMS based adaptive filtering is presented. For this, the input and the desired response signals are properly chosen in such a way that the filter output is the best least squared estimate of the original ECG signal. The proposed treatment exploits the modifications in the weight update formula and thus pushes up the speed over the respective LMS based realizations. Our simulations, however, confirm that the performance of the CSLMS is better than the LMS algorithm in terms of SNRI, MSE and misadjustment, this is shown in tables I and II. Hence CSLMS based adaptive noise canceler may be used in all practical applications.

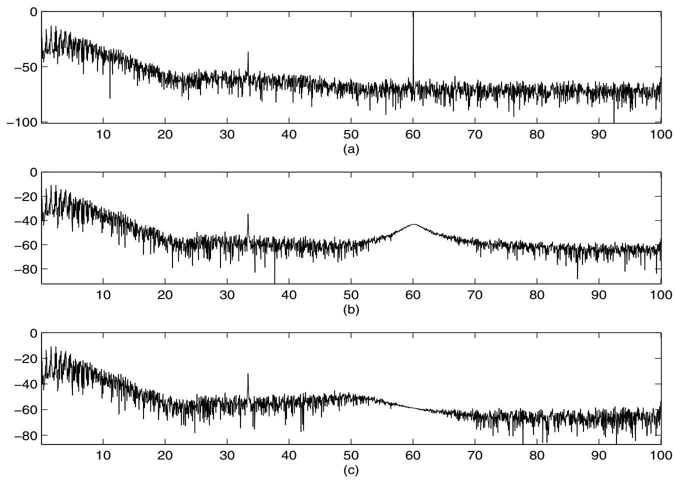

Fig. 5. (a) Frequency spectrum of ECG with PLI, (b) Frequency spectrum after filtering with LMS algorithm, (c) Frequency spectrum after filtering with CSLMS algorithm.

TABLE I

COMPARISON OF THE MMSE, MSE, EMSE AND M OF THE THREE EXAMINED ALGORITHMS.

| $\mu$ | $\sigma$ | Alg. | Min. MSE | MSE | Exce. MSE | Mis.Ad |
|---|---|---|---|---|---|---|
| 0.1 | 0.001 | LMS | 0.3243 | 0.1586 | -0.1655 | -0.5102 |
| | | NLMS | 0.3243 | 0.1525 | -0.1719 | -0.5299 |
| | | CSLMS | 0.3243 | 0.1450 | -0.1792 | -0.5527 |
| | 0.01 | LMS | 0.3222 | 0.1565 | -0.1657 | -0.5142 |
| | | NLMS | 0.3222 | 0.1502 | -0.1720 | -0.5337 |
| | | CSLMS | 0.3222 | 0.1429 | -0.1792 | -0.5563 |
| | 0.1 | LMS | 0.3047 | 0.1371 | -0.1676 | -0.5501 |
| | | NLMS | 0.3047 | 0.1317 | -0.1730 | -0.5679 |
| | | CSLMS | 0.3047 | 0.1251 | -0.1796 | -0.5895 |
| 0.5 | 0.001 | LMS | 0.3243 | 0.2404 | -0.0839 | -0.2586 |
| | | NLMS | 0.3243 | 0.1860 | -0.1383 | -0.4265 |
| | | ENLMS | 0.3243 | 0.1453 | -0.1789 | -0.5519 |
| | 0.01 | LMS | 0.3222 | 0.2370 | -0.0853 | -0.2646 |
| | | NLMS | 0.3222 | 0.1833 | -0.1389 | -0.4311 |
| | | ENLMS | 0.3222 | 0.1432 | -0.1789 | -0.5555 |
| | 0.1 | LMS | 0.3028 | 0.2056 | -0.0972 | -0.3210 |
| | | NLMS | 0.3028 | 0.1592 | -0.1436 | -0.4743 |
| | | ENLMS | 0.3028 | 0.1260 | -0.1779 | -0.5854 |

TABLE II

PERFORMANCE CONTRAST OF LMS AND CSLMS ALGORITHMS FOR THE CANCELATION OF ARTIFACTS (ALL VALUES ARE IN DECIBELS)

| Noise | Rec. No | SNRI after LMS | SNRI after ENLMS |
|---|---|---|---|
| BW | 101 | 2.2772 | 4.0204 |
| | 102 | 3.7013 | 4.9917 |
| | 103 | 3.3064 | 4.9690 |
| | 104 | 3.1798 | 4.9360 |
| | 105 | 3.2497 | 4.8894 |
| | Average | 3.1428 | 4.7613 |
| PLI | 101 | 6.1393 | 13.9129 |
| | 102 | 7.3513 | 14.0535 |
| | 103 | 5.7684 | 13.1728 |
| | 104 | 6.2568 | 13.3995 |
| | 105 | 5.9655 | 14.1440 |
| | Average | 6.3702 | 13.7365 |

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